NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2206

GRAPHICAL METHOD FOR OBTAINING FLOW FIELD IN TWO-DIMENSIONAL SUPERSONIC STREAM TO WHICH HEAT IS ADDED

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SUMMARY

A graphical method for obtaining the two-dimensional supersonic flow with moderate heat addition is presented. The basic equations were derived for flows in which the total temperature has a continuous variation with displacement in the direction of the flow. For flows at constant total temperatures before heat addition and with total temperature varying only in the direction of the streamlines, simplified forms of the basic equations were obtained from which working charts were plotted to facilitate construction of the flow. The working charts cover the range of Mach number from 1.3 to 5.0. The application of the method and the charts are illustrated by a simple example, giving the flow around a curved surface with addition of heat in an adjacent finite layer where the initial total temperature of the flow is the same on all streamlines and lines of constant total temperature in the heated portion of the flow lie approximately normal to the streamlines.

INTRODUCTION

Current interest in supersonic flow includes flow through gas turbines and in jet-engine exhausts in which heat release by continuing combustion of fuel introduced in an upstream combustion chamber modifies the flow. The general problem of the effect of heat addition on supersonic flows is of growing interest in the supersonic-flight and missile fields. The addition of heat to a gas flowing in a constant-area duct with a steady supersonic velocity has been investigated (reference 1). A theory for one-dimensional steady compressible fluid flow in ducts with heat addition is presented in reference 2. An extension of the method of characteristics (for example, see reference 3) for the graphical construction of irrotational supersonic flows was developed at the NACA Lewis laboratory and is presented herein for obtaining the two-dimensional supersonic flow to which heat is added. The method is restricted to flows in which the heat transfer between adjacent stream tubes can be neglected and entropy changes in the flow are due only to the addition of heat (shock-free flow).

The method of characteristics provides a construction of the irrotational flow field subdivided into zones in each of which the Mach number, and therefore all properties of the flow, are constant. When heat is added to the flow, further division of the flow is required to account for the variation of total temperature and total pressure throughout the field. 1387 In the rotational flow that results from the addition of heat, compression and expansion waves arise in the flow to maintain continuity of static pressure across adjacent streamlines. Because of the changes in total pressure and temperature and the generation of waves within, the flow streamlines and heat-addition reference lines must be plotted with the waves. The complexity of the graphical construction of the flow that results from heat addition and wave generation can be reduced by methods presented herein.

The equations developed as the basis of the proposed method apply to any mode of heat addition that gives a continuous variation of total temperature with displacement in the direction of the flow (but not necessarily continuous variation normal to the streamlines). For flows at constant total temperature before heat addition and with constant total temperature along lines approximately normal to the streamlines, simplified forms of the basic equations are obtained from which working charts have been plotted to facilitate the construction of the flow.

As an illustrative example, application of the proposed method is made to the problem of obtaining the heated supersonic-flow field adjacent to a curved wall. The initial total temperature of the flow is the same on all streamlines and lines of constant total temperature in the heated portion of the flow lie approximately normal to the streamlines.

SYMBOLS

The following symbols are used in this report:

c_p specific heat at constant pressure, herein assumed to be constant

c_v specific heat at constant volume

M Mach number

N square of Mach number

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P	total pressure
p	static pressure
Q	heat content of flow based on total temperature, $Q = c_p T$
R	gas constant
T	total temperature
t	static temperature
v	velocity
α	angle through which supersonic stream is turned to accelerate from $M = 1$ to $M > 1$
$\Delta\alpha$	wave strength expressed in terms of deflection produced in supersonic stream (positive value of $\Delta\alpha$ is that which increases static pressure)
β	Mach angle, $\sin^{-1} 1/M$
γ	ratio of specific heats, c_p/c_v
θ	angle between local velocity and direction of initial flow
ρ	density

Subscripts:

0	refers to initial conditions before addition of heat
1,2,3,...	designate particular sector of flow field bounded by waves, by heat-addition lines, or by both

METHOD

Basic Considerations

For the purposes of the method presented, the addition of heat to the flow in a stream tube is considered a two-step process. The stream tube is divided into short intervals along its length, over each of which a small change in heat content of the flow occurs. All the heat addition in each interval is considered to take place along a line at the middle of the interval. The heat addition is assumed to take place at constant stream-tube area. Following the heat addition, the streamlines change direction, with a corresponding change in stream-tube area to adjust the local static pressure as required by the adjacent flow. 1387

Governing Equations

The change in static pressure with a heat addition ΔQ at constant stream-tube area is obtained by application of the conservation-of-momentum and the continuity equations. For the constant-area case, these equations have the form

$$p + \rho v^2 = K_1 \tag{1}$$

$$\rho v = K_2 \tag{2}$$

where K_1 and K_2 are constants. From the equation of state for perfect gases,

$$p = \rho R t$$

and the equation for Mach number,

$$M = \frac{v}{\sqrt{\gamma R t}}$$

equations (1) and (2) become, respectively,

$$p(1+\gamma N) = K_1 \tag{3}$$

$$\frac{\gamma p^2 N}{Rt} = K_2{}^2 \tag{4}$$

where $N = M^2$. Differentiation of equations (3) and (4), written in the approximate-difference form, and substitution of $T = t\left(1 + \frac{\gamma-1}{2} N\right)$ yield, on the elimination of $\Delta N/N$ and $\Delta p/p$, successively,

$$\frac{\Delta p}{p} = \frac{\gamma N\left(1 + \frac{\gamma-1}{2} N\right)}{(N-1)} \frac{\Delta T}{T} = C \frac{\Delta Q}{Q} \tag{5}$$

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$$\frac{\Delta N}{N} = -\frac{(1+\gamma N)\left(1 + \frac{\gamma-1}{2} N\right)}{(N-1)} \frac{\Delta T}{T} = \frac{D}{N} \frac{\Delta Q}{Q} \tag{6}$$

where

$$C = \frac{\gamma N \left(1 + \frac{\gamma-1}{2} N\right)}{(N-1)} \tag{7a}$$

$$D = -\frac{N(1+\gamma N)\left(1 + \frac{\gamma-1}{2} N\right)}{N-1} \tag{7b}$$

$$Q = c_p T \tag{7c}$$

$$\frac{\Delta Q}{Q} = \frac{\Delta T}{T} \tag{7d}$$

Adjustment of the pressure following heat addition, as required by the adjacent flow, is accomplished by a wave of strength $\Delta\alpha$ generated in the flow at the point of adjustment shown in figure 1. If Δp is the required pressure change across the wave, the value is given by the simplified relation suitable for waves of small strength,

$$\Delta p = \frac{\gamma p N}{\sqrt{N-1}} \Delta\alpha = pA \ \Delta\alpha \tag{8}$$

where

$$A = \frac{\gamma N}{\sqrt{N-1}} \tag{8a}$$

and a positive value of $\Delta\alpha$ is that which produces an increase in static pressure. The sum of static-pressure changes resulting from heat addition at constant area, followed by pressure adjustment, is obtained from equations (5) and (8):

$$\frac{\Delta p}{p} = \frac{\gamma N \left(1 + \frac{\gamma-1}{2} N\right)}{(N-1)} \frac{\Delta Q}{Q} + \frac{\gamma N}{\sqrt{N-1}} \Delta\alpha = C \frac{\Delta Q}{Q} + A \ \Delta\alpha \tag{9}$$

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The Mach number change in the flow crossing the wave may be obtained by differentiating the well-known Prandtl-Meyer expression

$$-\alpha = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\left(\sqrt{\frac{\gamma-1}{\gamma+1}}\sqrt{N-1}\right) - \tan^{-1}\sqrt{N-1}$$

to obtain the expression for a small finite change in α

$$-\Delta\alpha = \frac{\Delta N}{2N}\frac{\sqrt{N-1}}{1+\frac{\gamma-1}{2}N} \tag{10a}$$

or

$$\frac{\Delta N}{N} = -\frac{2\left(1+\frac{\gamma-1}{2}N\right)}{\sqrt{N-1}}\Delta\alpha \tag{10b}$$

The total change in ΔN for the heat addition at constant area and subsequent pressure adjustment is obtained from equations (6) and (10b) as

$$\frac{\Delta N}{N} = -\frac{(1+\gamma N)\left(1+\frac{\gamma-1}{2}N\right)}{N-1}\frac{\Delta Q}{Q} - \frac{2\left(1+\frac{\gamma-1}{2}N\right)}{\sqrt{N-1}}\Delta\alpha \tag{10c}$$

Application of Governing Equations

Equations are now developed for four distinct cases of flow adjustment that arise in the solution of heated supersonic flows by the method of this report. To each case of flow adjustment corresponds a generated wave pattern that can be computed by application of equations (5) to (10b). Because the method of solution applies to any mode of heat addition that provides a continuous variation of temperature along a streamline, the heat content per unit mass of fluid Q may vary discontinuously with displacement across streamlines. For the special case in which the total temperature is constant on all streamlines upstream of the heat-addition region and lines of constant total temperature lie normal to the streamlines in the heat-addition zone, simplified expressions suitable for computing working charts are obtained for the waves generated in the four cases of flow adjustment.

The marked effect of small quantities of heat added to the flow, representing a change of several percent in total temperature or heat content Q, on the local properties of the flow requires that heat-addition reference lines be drawn in the flow
1387 to devide the flow into zones in which a change in heat content Q of only 2 or 3 percent occurs. These reference lines, drawn normal to the local streamlines for convenience, represent the interval center lines discussed in the previous section, at which all the heat added in the interval is concentrated.

Case I: wave generation at boundary between heated and unheated flow. - The first case considered is schematically illustrated in figure 1, which shows the flow adjustment that occurs along a streamline separating heated and unheated portions of the flow. The pressure equilibrium that exists across the bounding streamline before heat addition is restored after heat addition by a deflection of the flow accompanied by the generation of two waves of equal magnitude but opposite sign. Subscripts used in the following derivations refer to flow regions numbered in the figure. From equation (5), the pressure p_3 in the heated stream tube, after a head addition of ΔQ, is

$$p_3 = p_2 + p_2 \left[\frac{\gamma N\left(1 + \frac{\gamma-1}{2} N\right)}{N-1}\right]_2 \frac{(\Delta Q)_2}{Q_2} \tag{11a}$$

where $(\Delta Q)_2$ and Q_2 are associated with the flow in region 2. If $\Delta\alpha_1$ is the turn in the bounding streamline required to restore pressure equilibrium in the regions 4 and 5, then from equation (8),

$$p_4 = p_3 - p_3 \left(\frac{\gamma N}{\sqrt{N-1}}\right)_3 \Delta\alpha_1 \tag{11b}$$

$$p_5 = p_1 + p_1 \left(\frac{\gamma N}{\sqrt{N-1}}\right)_1 \Delta\alpha_1 \tag{11c}$$

If $\Delta\alpha_1$ is the wave strength associated with the flow having the value of $N = N_1$, then the wave separating regions 3 and 4 has the strength $-\Delta\alpha_1$. With the condition that the static pressures on both sides of the bounding streamline are equal after a heat addition of $(\Delta Q)_2$, $p_4 = p_5$, and with the substitution of equation (11a) in equation (11b), the following expression is obtained:

$$p_1 + p_1\left(\frac{\gamma N}{\sqrt{N-1}}\right)_1 \Delta\alpha_1 = p_2 + p_2\left[\frac{\gamma N\left(1 + \frac{\gamma-1}{2} N\right)}{N-1}\right]_2 \frac{(\Delta Q)_2}{Q_2} -$$

$$p_2\left\{1 + \left[\frac{\gamma N\left(1 + \frac{\gamma-1}{2} N\right)}{N-1}\right]_2 \frac{(\Delta Q)_2}{Q_2}\right\}\left(\frac{\gamma N}{\sqrt{N-1}}\right)_3 \Delta\alpha_1 \tag{11d}$$

With the substitution in equation (11d) of N_3 in terms of N_2 by means of equation (6), and from the relation $p_1 = p_2$, the solution of the equation (11d) for $\Delta\alpha_1$ in terms of known quantities is

$$\Delta\alpha_1 = \frac{C_2 \frac{(\Delta Q)_2}{Q_2}}{A_1 + \left[1 + C_2 \frac{(\Delta Q)_2}{Q_2}\right] \frac{\gamma F_2}{\sqrt{F_2 - 1}}} \tag{12}$$

where A, C, and D are defined by equations (8a), (7a), and (7b), respectively, and

$$F = N + D \frac{\Delta Q}{Q} \tag{12a}$$

The expression for $\Delta\alpha_1$ (equation (12)) can be written as a function of N_1/N_2 and N_2 that is convenient for presentation in chart form. The chart and its uses are presented when the complete graphical method is considered.

The pressure change p_4-p_2 is obtained by means of equations (11c) and (12) as

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$$p_4-p_2 = p_5-p_1 = p_1\left\{\frac{A_1C_2\left[\frac{(\Delta Q)_2}{Q_2}\right]}{A_1 + \left[1 + C_2\frac{(\Delta Q)_2}{Q_2}\right]\frac{\gamma F_2}{\sqrt{F_2-1}}}\right\} \tag{13}$$

Case II: interaction between wave and heat-addition zone. - In the portion of the flow schematically depicted in figure 2, a wave crosses a heat-addition zone at point P. The flow above the streamline through P crosses the incident wave before encountering the heat-addition zone. The flow in region 2 below the streamline through P crosses the heat-addition line first. In order to equalize the pressure in regions 4 and 5, the incident wave, on transmission through the heat-addition zone at P, results in a modification of the incident wave to a strength $\Delta\alpha_k$ and a generation of a wave of strength $\Delta\alpha_j$. The pressure in region 7 is given (from equation (8)) as

$$p_7 = p_1 + p_1\left(\frac{\gamma N}{\sqrt{N-1}}\right)_1\Delta\alpha \tag{14a}$$

The corresponding change in Mach number may be obtained by use of equation (10b). The pressure in region 6 is obtained by means of equation (5) in terms of the flow quantities in region 7;

$$p_6 = p_7 + p_7\left[\frac{\gamma N\left(1 + \frac{\gamma-1}{2}N\right)}{N-1}\right]_7\frac{(\Delta Q)_1}{Q_1} \tag{14b}$$

where ΔQ_1 and Q_1 are associated with the flow from region 1.

In order for the flows in regions 4 and 5 to have the same direction, the strength of the wave $\Delta\alpha_j$, arising at point P, is determined as

$$\Delta\alpha_j = \Delta\alpha - \Delta\alpha_k \tag{15}$$

The pressure in region 5 is given in terms of the flow quantities in region 6 as

$$p_5 = p_6 + p_6 \left(\frac{\gamma N}{\sqrt{N-1}}\right)_6 (\Delta\alpha - \Delta\alpha_k) \tag{14c}$$

By substituting successively for the pressure p_6 and p_7, the pressure in region 5 may be obtained in the following form:

$$p_5 = p_1 \left[1 + \left(\frac{\gamma N}{\sqrt{N-1}}\right)_1 \Delta\alpha\right] \left\{1 + \left[\frac{\gamma N \left(1 + \frac{\gamma-1}{2} N\right)}{N-1}\right]_7 \frac{(\Delta Q)_1}{Q_1}\right\} \left[1 + \left(\frac{\gamma N}{\sqrt{N-1}}\right)_6 (\Delta\alpha - \Delta\alpha_k)\right] \tag{14d}$$

When equations (10b) and (6) are used to obtain the values of N at regions 7 and 6, the equation (14d) takes the form

$$p_5 = p_1 \left(1 + A_1 \Delta\alpha\right) \left(1 + \left\{\frac{\gamma(N+B\Delta\alpha)\left[1 + \frac{\gamma-1}{2}(N+B\Delta\alpha)\right]}{N+B\Delta\alpha-1}\right\}_1 \frac{(\Delta Q)_1}{Q_1}\right)$$

$$\left[1 + \left(\frac{\gamma\left\{N+B\Delta\alpha - \frac{(N+B\Delta\alpha)(1+\gamma N+\gamma B\Delta\alpha)\left[1+\frac{\gamma-1}{2}(N+B\Delta\alpha)\right] \frac{(\Delta Q)_1}{Q_1}}{N+B\Delta\alpha - 1}\right\}}{\sqrt{N+B\Delta\alpha-1 - \frac{(N+B\Delta\alpha)(1+\gamma N+\gamma B\Delta\alpha)\left[1+\frac{\gamma-1}{2}(N+B\Delta\alpha)\right] \frac{(\Delta Q)_1}{Q_1}}{N+B\Delta\alpha - 1}}}\right)_1 (\Delta\alpha - \Delta\alpha_k)\right] \tag{14e}$$

where

$$B = -\frac{2N\left(1 + \frac{\gamma-1}{2} N\right)}{\sqrt{N-1}} \tag{16}$$

Equation (16) may be more simply written, by means of the substitution

$$E = N + B\,\Delta\alpha \tag{14f}$$

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$$p_5 = p_1 \left(1 + A_1 \Delta\alpha\right) \left\{1 + \left[\frac{\gamma E\left(1 + \frac{\gamma-1}{2} E\right)}{E-1}\right]_1 \frac{(\Delta Q)_1}{Q_1}\right\}$$

$$\left(1 + \left\{\frac{\gamma\left[E - \frac{E(1+\gamma E)\left(1 + \frac{\gamma-1}{2} E\right)\frac{(\Delta Q)_1}{Q_1}}{E-1}\right]}{\sqrt{E-1 - \frac{E(1+\gamma E)\left(1 + \frac{\gamma-1}{2} E\right)\frac{(\Delta Q)_1}{Q_1}}{E-1}}}\right\}_1 (\Delta\alpha - \Delta\alpha_k)\right) \tag{14g}$$

In a similar manner, the pressure in region 4 may be obtained as

$$p_4 = p_2\left\{1 + C_2 \frac{(\Delta Q)_2}{Q_2} + \gamma\left[1 + C_2 \frac{(\Delta Q)_2}{Q_2}\right]\frac{F_2}{\sqrt{F_2-1}} \Delta\alpha_k\right\} \tag{14h}$$

where C and F are functions of N as defined in equations (7a) and (12a), respectively. With the use of $p_1 = p_2$ and $p_5 = p_4$, equations (14g) and (14h) may be solved for $\Delta\alpha_k$ as a function of the known quantities N_1, N_2, $(\Delta Q)_1/Q_1$, $(\Delta Q)_2/Q_2$, and $\Delta\alpha$:

$$\Delta\alpha_k = \frac{Z_1 - 1 + Y_1 Z_1 \Delta\alpha - C_2 \frac{(\Delta Q)_2}{Q_2}}{Y_1 Z_1 + W_2} \tag{17a}$$

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where

$$
\left.
\begin{aligned}
Y_1 &= \frac{\gamma\left[E - \dfrac{E(1+\gamma E)\left(1+\dfrac{\gamma-1}{2}E\right)\dfrac{(\Delta Q)_1}{Q_1}}{E-1}\right]_1}{\left[\sqrt{E-1-\dfrac{E(1+\gamma E)\left(1+\dfrac{\gamma-1}{2}E\right)\dfrac{(\Delta Q)_1}{Q_1}}{E-1}}\right]_1} \\
Z_1 &= \left(1 + A_1\,\Delta\alpha_1\right)\left\{1 + \left[\frac{\gamma E\left(1+\dfrac{\gamma-1}{2}E\right)\dfrac{(\Delta Q)_1}{Q_1}}{E-1}\right]_1\right\}_1 \\
W_2 &= \left\{\gamma\left[1 + C_2\,\frac{(\Delta Q)_2}{Q_2}\right]\left(\frac{F}{\sqrt{F-1}}\right)_2\right\}_2
\end{aligned}
\right\} \tag{18}
$$

Where the initial total temperature is constant and the heat addition is considered constant along lines normal to the local streamlines, $Q_1 = Q_2 = Q$ and $(\Delta Q)_1 = (\Delta Q)_2 = \Delta Q$. Therefore, with $N_1 = N_2 = N$, equation (17a) is written in terms of N, $\Delta Q/Q$, and $\Delta\alpha$ as

$$\Delta\alpha_k = \frac{Z-1 + YZ\,\Delta\alpha - C\,\dfrac{\Delta Q}{Q}}{YZ + W} \tag{17b}$$

where Y, Z, and W are evaluated as given by equations (18), with $Q_1 = Q_2 = Q$, $(\Delta Q)_1 = (\Delta Q)_2 = \Delta Q$, and $N_1 = N_2 = N$. The pressure change may be written as

$$\frac{p_5-p_1}{p_1} = \frac{p_4-p_1}{p_1} = C_2\,\frac{(\Delta Q)_2}{Q_2} + W_2\left[\frac{Z_1-1 + Y_1Z_1\,\Delta\alpha - C_2\,\dfrac{(\Delta Q)_2}{Q_2}}{Y_1Z_1 + W_2}\right] \tag{19}$$

Case III: wave propagation across adjacent stream tubes of differing total pressure. - A wave passing through adjacent stream tubes of differing total pressure produces unequal changes of static pressure in the stream tubes. In order to maintain continuity of static pressure and flow direction downstream of the wave, the wave is modified in strength and another wave is generated at the streamlines separating two adjacent stream tubes of differing total pressure. The situation schematically illustrated in figure 3 is considered: A wave of strength $\Delta\alpha$ is shown crossing the boundary between regions 1 and 2 in which the flow is parallel and at the same static pressure upstream of the wave, but differs in Mach number. If $\Delta\alpha_t$ is the strength of the wave transmitted through the bounding streamline and $\Delta\alpha_r$ the strength of the wave generated at the bounding streamline, then

$$p_5 = p_1 + p_1\left(\frac{\gamma N}{\sqrt{N-1}}\right)_1 \Delta\alpha_t \tag{20a}$$

$$p_3 = p_2 + p_2\left(\frac{\gamma N}{\sqrt{N-1}}\right)_2 \Delta\alpha \tag{20b}$$

The generated wave $\Delta\alpha_r$ will produce a change of pressure that is given by

$$p_4 = p_3 + p_3\left(\frac{\gamma N}{\sqrt{N-1}}\right)_3 \Delta\alpha_r \tag{20c}$$

In order for the flow in adjacent stream tubes to have the same direction,

$$\Delta\alpha_r = \Delta\alpha - \Delta\alpha_t \tag{21}$$

The requirement that the pressures in regions 4 and 5 be equal provides, by means of equations (10b), (20), and (21),

$$p_1(1+A_1\ \Delta\alpha_t) = p_2\left[(1+A_2\ \Delta\alpha) + (1+A_2\ \Delta\alpha)\left(\frac{\gamma E}{\sqrt{E-1}}\right)_2 (\Delta\alpha-\Delta\alpha_t)\right] \tag{22}$$

Inasmuch as $p_1 = p_2$, solving for $\Delta\alpha_t/\Delta\alpha$ gives 1387

$$\frac{\Delta\alpha_t}{\Delta\alpha} = \frac{A_2 + \left(\frac{\gamma E}{\sqrt{E-1}}\right)_2 (1+A_2\ \Delta\alpha)}{A_1 + \left(\frac{\gamma E}{\sqrt{E-1}}\right)_2 (1+A_2\ \Delta\alpha)} \tag{23a}$$

Where the strength of the incident wave $\Delta\alpha$ can be assumed to be small $(|\Delta\alpha| \approx 1.00^\circ)$, calculations have shown that the pressure and the Mach number changes from region 2 to region 4 can be considered to result from a combined wave of strength $\Delta\alpha + \Delta\alpha_r$ acting on the flow in region 2. Equation (23a) then is of the simpler form

$$\frac{\Delta\alpha_t}{\Delta\alpha} = \frac{2}{\frac{A_1}{A_2} + 1} = 1 - \frac{\Delta\alpha_r}{\Delta\alpha} \tag{23b}$$

<u>Case IV: heat addition across adjacent stream tubes of differing total pressure.</u> - The difference in the change in static pressure produced by additions of heat to adjacent stream tubes of differing total pressure is compensated by waves of equal intensity and opposite sign generated at the streamline separating the two stream tubes, as illustrated in figure 4. If $(\Delta Q)_1/Q_1$ and $(\Delta Q)_2/Q_2$ are the values associated with the flow from the regions 1 and 2, respectively, the pressure in the regions 6 and 3 are obtained from equation (5) as

$$p_6 = p_1 + p_1\left[\frac{\gamma N\left(1 + \frac{\gamma-1}{2} N\right)}{N - 1}\right]_1 \frac{(\Delta Q)_1}{Q_1} \tag{24a}$$

$$p_3 = p_2 + p_2\left[\frac{\gamma N\left(1 + \frac{\gamma-1}{2} N\right)}{N - 1}\right]_2 \frac{(\Delta Q)_2}{Q_2} \tag{24b}$$

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If $\Delta\alpha_1$ is the wave associated with the flow having the value of $N = N_1$, then the wave separating regions 3 and 4 has the strength $-\Delta\alpha_1$. The pressures in the regions 5 and 4 in terms of the flow quantities in the regions 6 and 3, respectively, are

$$p_5 = p_6 + p_6\left(\frac{\gamma N}{\sqrt{N-1}}\right)_6 \Delta\alpha_1 \tag{24c}$$

$$p_4 = p_3 - p_3\left(\frac{\gamma N}{\sqrt{N-1}}\right)_3 \Delta\alpha_1 \tag{24d}$$

Because $p_5 = p_4$, the substitution of equations (24a) and (24b) in equations (24c) and (24d), respectively, yields

$$p_1\left\{\left[1+C_1 \frac{(\Delta Q)_1}{Q_1}\right] + \left[1+C_1 \frac{(\Delta Q)_1}{Q_1}\right] \frac{\gamma F_1}{\sqrt{F_1-1}} \Delta\alpha_1\right\}$$

$$= p_2\left\{\left[1+C_2 \frac{(\Delta Q)_2}{Q_2}\right] - \left[1+C_2 \frac{(\Delta Q)_2}{Q_2}\right] \frac{\gamma F_2}{\sqrt{F_2-1}} \Delta\alpha_1\right\} \tag{25}$$

where C and F are functions of N previously defined (equations (7a) and (12a), respectively). Because $p_1 = p_2$, equation (25), solved for $\Delta\alpha_1$, yields

$$\Delta\alpha_1 = -\frac{C_1 \dfrac{(\Delta Q)_1}{Q_1} - C_2 \dfrac{(\Delta Q)_2}{Q_2}}{\left[1 + C_2 \dfrac{(\Delta Q)_2}{Q_2}\right]\dfrac{\gamma F_2}{\sqrt{F_2-1}} + \left[1 + C_1 \dfrac{(\Delta Q)_1}{Q_1}\right]\dfrac{\gamma F_1}{\sqrt{F_1-1}}} \tag{26a}$$

If the initial total temperature is constant and the heat addition is constant along lines normal to the local streamlines, $(\Delta Q)_1 = (\Delta Q)_2 = \Delta Q$ and $Q_1 = Q_2 = Q$. Then equation (26a) becomes

$$\Delta\alpha_1 = - \frac{(C_1 - C_2)\ \frac{\Delta Q}{Q}}{\left(1 + C_2 \frac{\Delta Q}{Q}\right)\frac{\gamma F_2}{\sqrt{F_2 - 1}} + \left(1 + C_1 \frac{\Delta Q}{Q}\right)\frac{\gamma F_1}{\sqrt{F_1 - 1}}} \tag{26b}$$

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Working charts. - By means of equations (5), (6), (12), (17b), (23b), and (26b), and with $\gamma = 1.4$, charts were obtained to facilitate the graphical construction of heated supersonic flows where the initial total temperature is the same on all streamlines and the lines of constant total temperature in the heated portion of the flow are normal to the local streamlines. These conditions are approximated in ducts and channels having an initially uniform supersonic flow in which heat release by chemical reaction is occurring in the body of the flow.

The exact differential form of equation (6), giving the change in N with a heat addition of $\Delta Q/Q$ at constant area, has been integrated and the results are given in figure 5 for $\Delta Q/Q$ of 0.02 and 0.03 with $\Delta N = N_1 - N_2$ as the ordinate and N_1 as the abscissa. The static-pressure change and the total-pressure ratio across a heat addition have been calculated by integrating the exact differential form of equation (5) and using the values of N_2 obtained from figure 5 for a given $\Delta Q/Q$. The variation of the static-pressure change and the total-pressure ratio across a heat-addition zone with N_1, the value of N preceding heat addition are given in figures 6 and 7, respectively.

The value of $\Delta\alpha_1$ of case I as a function of N_2, with N_1/N_2 as a parameter, is presented in figures 8(a) and 8(b) by means of equation (12) for $\Delta Q/Q$ of 0.02 and 0.03, respectively. The value of $\Delta\alpha_1$ in case II as a function of N_1 with various values of $\Delta\alpha$ is presented in figure 9 by means of equation (17b) for $\Delta Q/Q$ of 0.02 and 0.03 for both expansion and compression waves. The ratios $\Delta\alpha_t/\Delta\alpha$ and $\Delta\alpha_r/\Delta\alpha$ of case III are calculated from equation (23b) and presented in figure 10 as functions of A_1/A_2. (Equation (23b), which is an approximate form of equation (23a), gives sufficient accuracy in the range $N = 1.5$ to $N = 25.0$ for small values of $\Delta\alpha$.) For $|\Delta\alpha| \approx 1.00^\circ$, the use of the chart of figure 10 gives values of the modified and generated waves, which have less than 0.5-percent deviation from the values obtained from the more exact equation (23a).

The variation of A (equation (8a)) with N is shown in figure 11. The value of $\Delta\alpha_1$ of case IV, calculated from equation (26b), is presented in figures 12(a) to (12d) as a function of

N_1 with various values of N_2. Figures 12(a) and 12(b) give the values for $\Delta\alpha_1$ of case IV with $\Delta Q/Q = 0.02$, and figures 12(c) and 12(d) give the values for $\Delta\alpha_1$ of case IV with $\Delta Q/Q = 0.03$. For a given $\Delta Q/Q$, the curve of $\Delta\alpha_1$ against N_1 is coincident for $N_2 = 3.0$ and $N_2 = 4.0$ and very similar for $3.0 < N_2 < 4.0$. Figures 13 and 14 show plots of N against α, the Prandtl-Meyer turning angle, and N against β, the Mach angle, respectively, with expanded scales that are convenient for graphical plotting.

COMPLETE GRAPHICAL METHOD

The initial layout of the graphical construction for heated supersonic flows begins as for an irrotational supersonic flow. Curved walls bounding the flow are approximated by chord lines of small angles of inclination relative to adjacent chord lines. If the initial upstream flow is rotational, the flow is divided into stream tubes in each of which the Mach number is considered to have the value on the streamline through the middle of the stream tube. The plot of the flow proceeds by successive application of the four cases for which equations were developed in the preceding section of the report. It is assumed that in any problem the variation in total temperature through the heated portion of the flow is sufficiently well known for the accuracy desired in the construction of the flow. As the flow construction proceeds, heat-addition reference lines are drawn normal to the local streamlines. For flows in which the lines of constant total temperature also lie normal to the streamlines, the heat-addition reference lines are also lines of constant total temperature. In the interval between consecutive reference lines, the fractional heat addition to the flow $\Delta Q/Q$ should be equal to or less than 0.03 for values of N from 1.5 to 16.0 in order for the equations representing the effect of heat addition on the flow to apply with satisfactory accuracy. For values of N greater than 16.0, smaller values of $\Delta Q/Q$ are to be used, (for example, for N of 25.0, a satisfactory value for $\Delta Q/Q$ is 0.02). When the total temperature is not constant along reference lines, the flow is divided into stream tubes, in each of which the total temperature is assumed constant and equal to the average of the values at the bounding streamlines. A sufficient number of stream tubes are chosen to approximate the variation of total temperature normal to the streamlines.

The application of the flow-construction method considered is essentially the same for flows in which the total temperature is constant or varies along lines normal to the streamlines. In order to illustrate the application of the working charts, an example is given in which the total temperature is constant on lines normal to the streamlines.

This example, shown in figure 15, presents the contruction of a heated supersonic flow adjacent to a curved wall. The flow, initially uniform at $M_0 = 4.0$, is bounded by a single wall curved through 4° on a circular arc between points A and B and the unheated flow extending to infinity. The streamline bounding the heated portion of the flow goes through points E and F. A quantity of heat $\Delta Q/Q = 0.03$ is added at regularly spaced intervals corresponding to 1° turn of the bounding wall. In any interval n, $Q = Q_0(1 + \Delta Q/Q)^n$, where Q_0 is the value of Q before heat addition. Waves arising or reflected from curved walls bounding the flow are handled in the manner employed in two dimensional irrotational flows.

The curved wall is approximated by chord lines of increasing inclination to the initial flow direction in increments of 1°. The first wave is generated at the wall at point C with the strength $\Delta\alpha = -1.0^{\circ}$. The corresponding Mach number change and Mach angle β imposed on the flow by the wave are obtained from figures 13 and 14, respectively, which give, for $N_1 = 16.0$ and $\Delta\alpha = -1.0^{\circ}$, $N_2 = 16.62$ and $\beta = 14.48^{\circ}$. The value of N and the inclination of the flow to the initial flow direction are recorded in figure 15 as N/θ, which in this case is $16.62/-1.000^{\circ}$. Negative values of θ indicate a clockwise turn of the flow where the initial direction of the flow is from left to right. At the intersection of the wave and the first heat-addition reference line at D (fig. 15), the situation classified as case II occurs. The generated and transmitted waves are obtained from figure 9(c) and equation (15). With $\Delta\alpha = -1.00^{\circ}$, $N_1 = 16.00$, and $\Delta Q/Q = 0.03$, $\Delta\alpha_j$ is equal to 0.034° and $\Delta\alpha_k = -1.034^{\circ}$. The change in the value of N across the heat-addition reference line is obtained from figure 5, with subscripts 1 and 2 representing the upstream and downstream flow, respectively. The change in the value of N across each wave is obtained from figure 13. The values for N and θ are shown in the appropriate sections of the flow (fig. 15) and $\Delta\alpha_j$ and $\Delta\alpha_k$ are recorded on the corresponding waves. The streamline through point D (fig. 15) is turned through an angle $\Delta\alpha_k$. The flows above and below this streamline have different total pressures and the streamline DH is drawn in the plot of the flow to demark this difference.

Along the streamline bounding the heated and unheated flow, two waves of equal magnitude but opposite sign are generated at the point of intersection E (fig. 15), with a heat-addition reference line corresponding to the situation classified as case I. The value of $\Delta\alpha_1$ is obtained from figure 8(b) for $N_1/N_2 = 16/16 = 1.00$, as $\Delta\alpha_1 = 0.899$. The change in N of the unheated flow is obtained from figure 13 for $N = 16$ and $\Delta\alpha = \Delta\alpha_1 = 0.899$. The change in N of the heated flow across the heat-addition zone is obtained from figure 5 for $N_1 = 16$ and $\Delta Q/Q = 0.03$, as $N = 13.33$, and across the wave $\Delta\alpha = -\Delta\alpha_1 = -0.899$ from figure 13 for $N_1 = 13.33$ and $\Delta\alpha = -0.899$ as $N = 13.78$. The flow zones involved are labeled with the appropriate values of N and θ and the strength of the waves are indicated as well. It should be observed that a positive (compression) wave extending from the upper left to the lower right will produce a clockwise turning of the flow, as does a negative (expansion) wave extending from the lower left to the upper right (for example, point D). Likewise, a counterclockwise turn of the flow is produced by both a positive wave extending from the lower left to the upper right and a negative wave from the upper left to the lower right (for example, point E).

At point F in figure 15, a wave is incident on a streamline separating regions of differing total pressure, which corresponds to case III. An enlarged figure of the zone in the neighborhood of point F is shown in figure 16(a). The required values of A_1 and A_2 are obtained from figure 11 for $N_1 = 14.55$ and $N_2 = 10.59$, giving $A_1 = 5.534$, $A_2 = 4.788$, and $A_1/A_2 = 1.156$. By using figure 10, the corresponding values of the transmitted and reflected waves $\Delta\alpha_t$ and $\Delta\alpha_r$ (fig. 3) may be obtained as -0.988° and -0.077°, respectively, for $\Delta\alpha = -1.065^\circ$. The change in N across the various waves is obtained from figure 13 for the corresponding initial values of N. The Mach angle between the streamline and the waves is obtained from figure 14 as the angle β. It should be noted that whenever A_1 and A_2 are very nearly equal, the value of $\Delta\alpha_r$ is so small that it can be assumed to be negligible and $\Delta\alpha_t$ is then equal to $\Delta\alpha$ (fig. 10).

Waves generated by the addition of heat to adjacent flows of different total pressure, considered as case IV (fig. 4), are shown in figure 15 at point G and are illustrated in detail in figure 16(b). The strengths of the generated waves, which are equal in magnitude but opposite in sign, are obtained from figure 12, depending on the value of $\Delta Q/Q$ and N_2. For the case cited, $N_1 = 10.52$, $N_2 = 10.39$,

$\Delta Q/Q = 0.03$, and $\Delta\alpha_1$ is evaluated as -0.0050. In general, the value of $\Delta\alpha_1$ is very small unless N_1 and N_2 differ appreciably. The change in the value of N across the heat-addition zone and the waves are obtained successively from figures 5 and 13 for the appropriate local values of N, and the corresponding values of β are obtained from figure 14. 1387

An enlargement of the region near point H in figure 15, in which the combination of cases II and IV occurs, is presented in figure 16(c). The wave $\Delta\alpha = -0.966$ crosses the heat-addition reference line at point H, as does the streamline that separates two regions of differing total pressure. From the graphical solution of case II, the values of the transmitted and the generated waves are $\Delta\alpha_k = -0.995$ and $\Delta\alpha_j = 0.029$, respectively. For case IV, the values of the generated waves are $\Delta\alpha_{upper} = -0.002$ and $\Delta\alpha_{lower} = 0.002$. Therefore, the values of the transmitted and the generated waves of case II and case IV are summed algebraically and plotted as -0.997 and 0.031, respectively, as shown in figure 16(c).

It is evident that the number of waves and streamlines separating regions in the flow increases as the plot of the flow develops. When it becomes desirable to limit the number of waves and streamlines that must be plotted, several devices may be used for combining waves and reducing the number of streamlines separating flows of differing total pressure. A typical example of the reduction in the number of waves by combination of a weak and an adjacent strong wave is shown in the region about I in figure 15, wherein weak waves are designated by short dashed lines. Figure 17(a) presents an enlargement of the portion of figure 15 near point I. The wave $\Delta\alpha = 0.030^\circ$ is combined with the wave $\Delta\alpha = -0.925^\circ$ by using algebraic addition of the wave strengths to give a resulting wave of strength $\Delta\alpha = -0.895^\circ$. This device is generally employed at the heat-addition reference line because every wave that crosses a heat-addition line generates an additional wave of minor strength. Figure 17(b), which gives an enlarged view of figure 15 near point J, depicts a similar situation where the incoming minor wave $\Delta\alpha = 0.034^\circ$ is combined with the wave $\Delta\alpha = -1.00^\circ$ arising at the wall boundary. The wave that is drawn downstream from point J at the wall boundary is equal to $-.0966^\circ$, which represents the algebraic sum of both wave strengths. This method of combining waves avoids the accumulated error that would result from neglecting the minor waves and introduces only minor local errors in the flow.

A method for reducing the number of streamlines separating regions of differing total pressures is presented in figures 17(c) and 17(d), which show an enlarged portion of figure 15 near point K. In figure 17(c), the streamlines E and F, if drawn past waves U and V, would separate regions of different total pressures but of the same static pressure. A single streamline H replaces E and F (fig. 17(d)). The streamline H is positioned according to the equation

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$$(N_2-N_1)d = (N_3-N_2)e \tag{27}$$

The flow zone N_2 is eliminated. This procedure may somewhat distort the rotational properties of the flow; also, when the differences in N become large, the accuracy of the resulting plot may be impaired.

If the number of strong waves that arise at the fixed or free-stream boundaries becomes very great, the method illustrated in figure 18 can be used at the expense of accuracy in the plot of the flow to reduce the plotting work. Along a line immediately upstream of a heat-addition zone, a break is made in the plot of the flow. In the space provided by the break, a simplified form of the flow is substituted for the upstream flow. In figure 18 the letters A, B, C, D designate waves immediately upstream of a heat-addition zone. In each region of constant total pressure, such as that enclosed by streamlines R and S, waves of the same family as A and C are combined by adding their strengths. The combined wave is located at the weighted mean distance between wave A and C, $Aa = Cc$, when A and C are taken to represent the absolute magnitude of the strengths of the corresponding waves.

The static pressure at any point in the flow field may be obtained from the relation

$$\frac{p}{P} = \left(1 + \frac{\gamma-1}{2} N\right)^{-\frac{\gamma}{\gamma-1}}$$

where P and N are the local values of the total pressure and the Mach number squared, respectively. The local value of N appears on the graphical plot. Because the total pressure changes with heat addition in a manner that depends on the variation of N along each streamline, the local value of the total pressure must be obtained by tracing the changes in the total pressure along the particular streamline passing through the given point. In order to obtain the change in the total pressure across each heat addition of $\Delta Q/Q$, figure 7 or the relation

$$\frac{P_2}{P_1} = \frac{(1 + \gamma N_1)}{(1 + \gamma N_2)} \left[\frac{\left(1 + \frac{\gamma-1}{2} N_1\right)}{\left(1 + \frac{\gamma-1}{2} N_2\right)} \right]^{-\frac{\gamma}{\gamma-1}}$$ 1387

may be used, where the subscripts 1 and 2 refer to the regions upstream and downstream of a heat-addition reference line on the graphical plot of the flow field.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, May 31, 1950.

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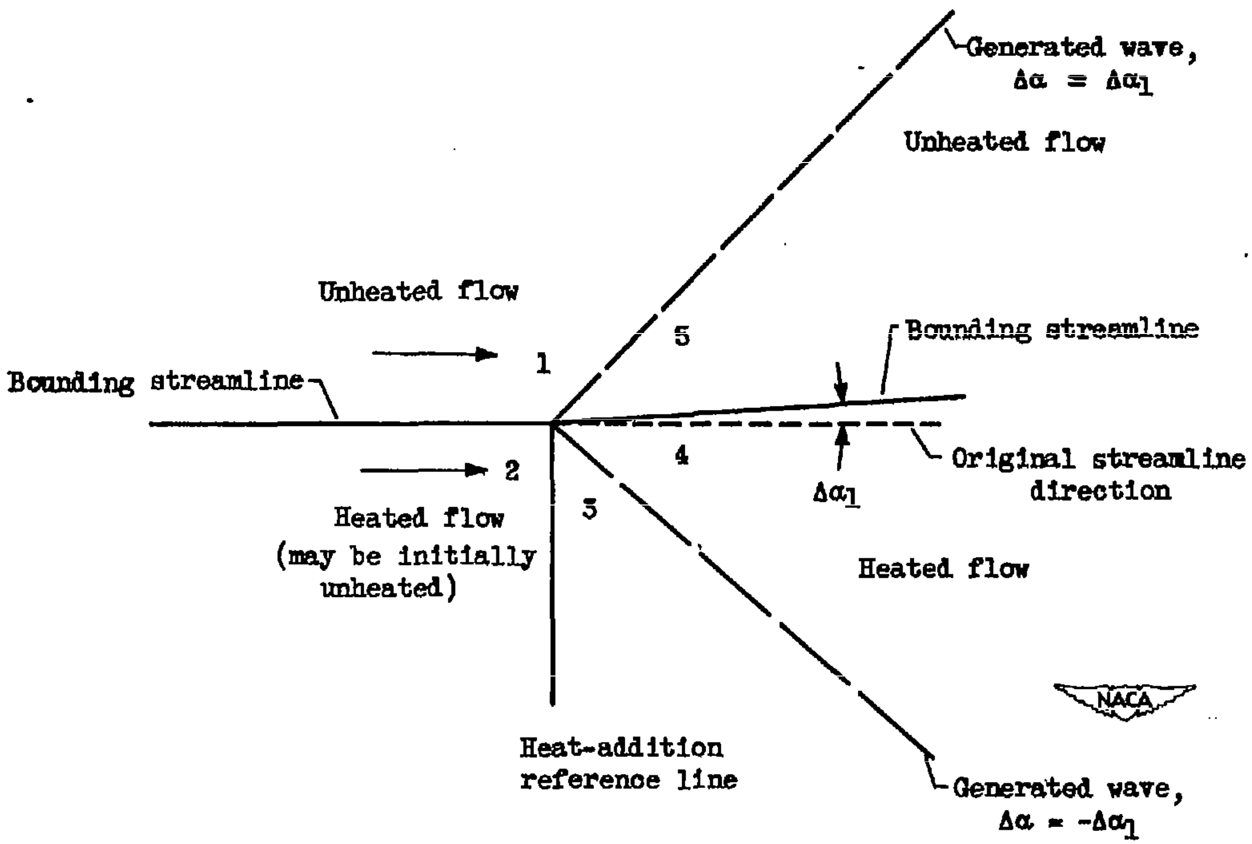


Figure 1. - Schematic representation of wave generation at boundary between unheated and heated flow; case I.

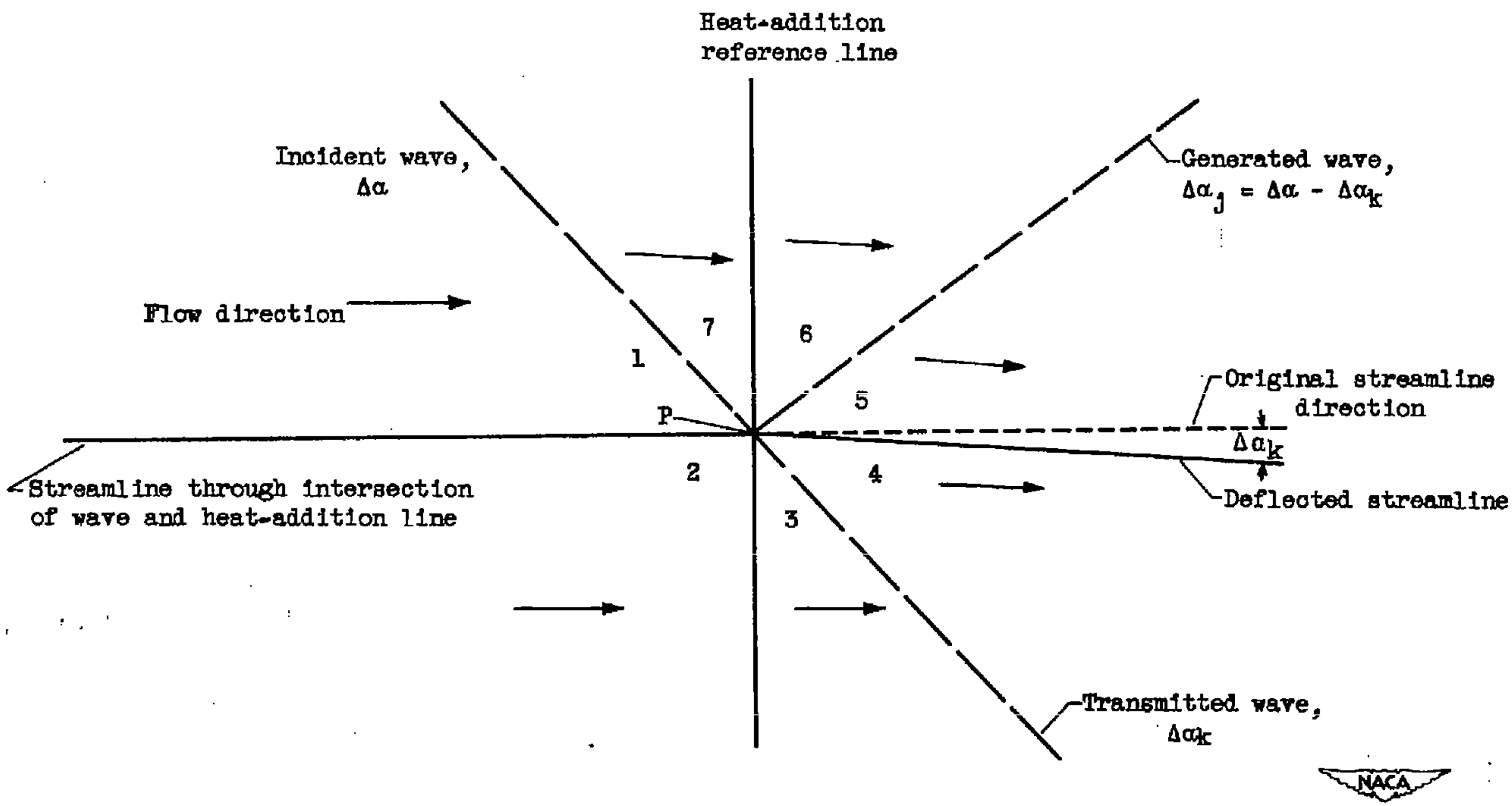


Figure 2. - Schematic representation of wave propagation across heat-addition reference line; case II.

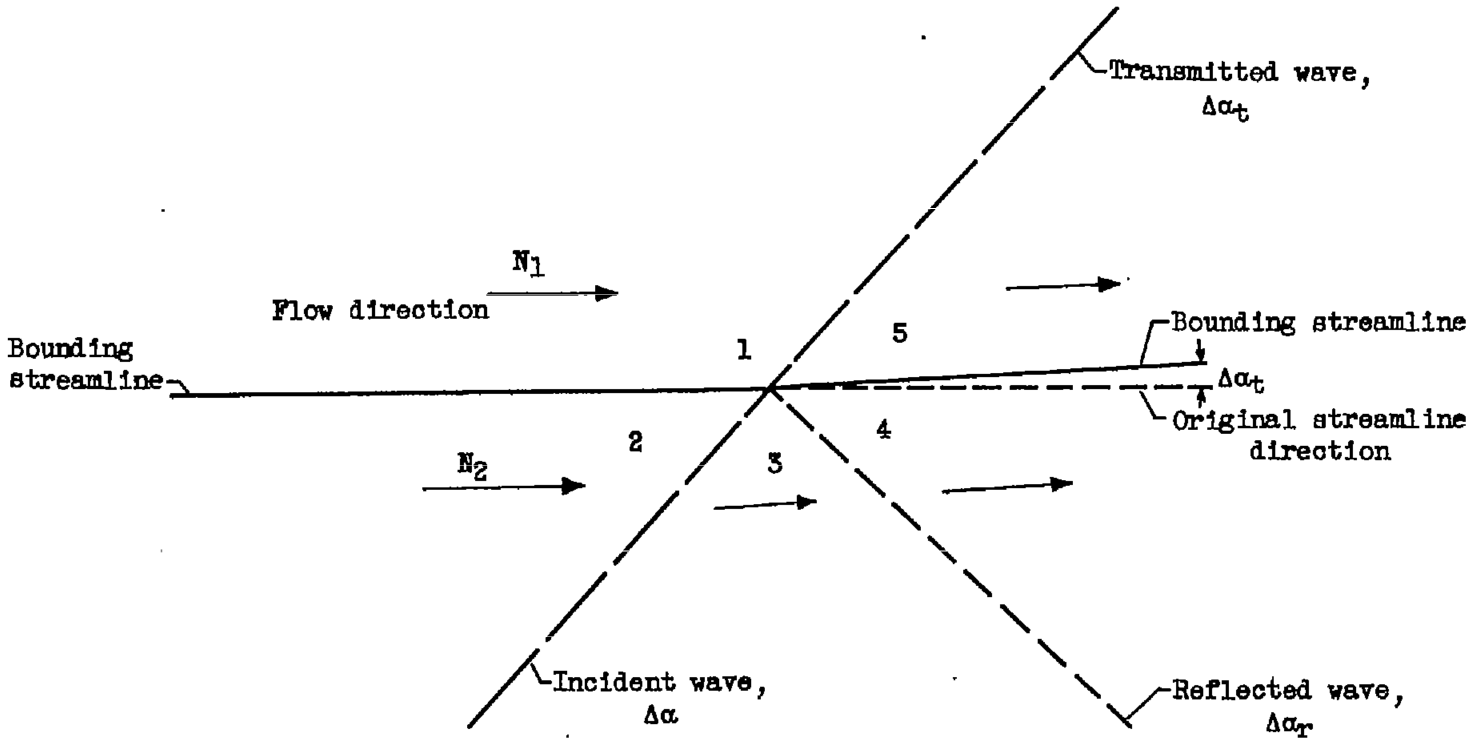


Figure 3. - Schematic representation of wave propagation across adjacent stream tubes of differing total pressure; case III.

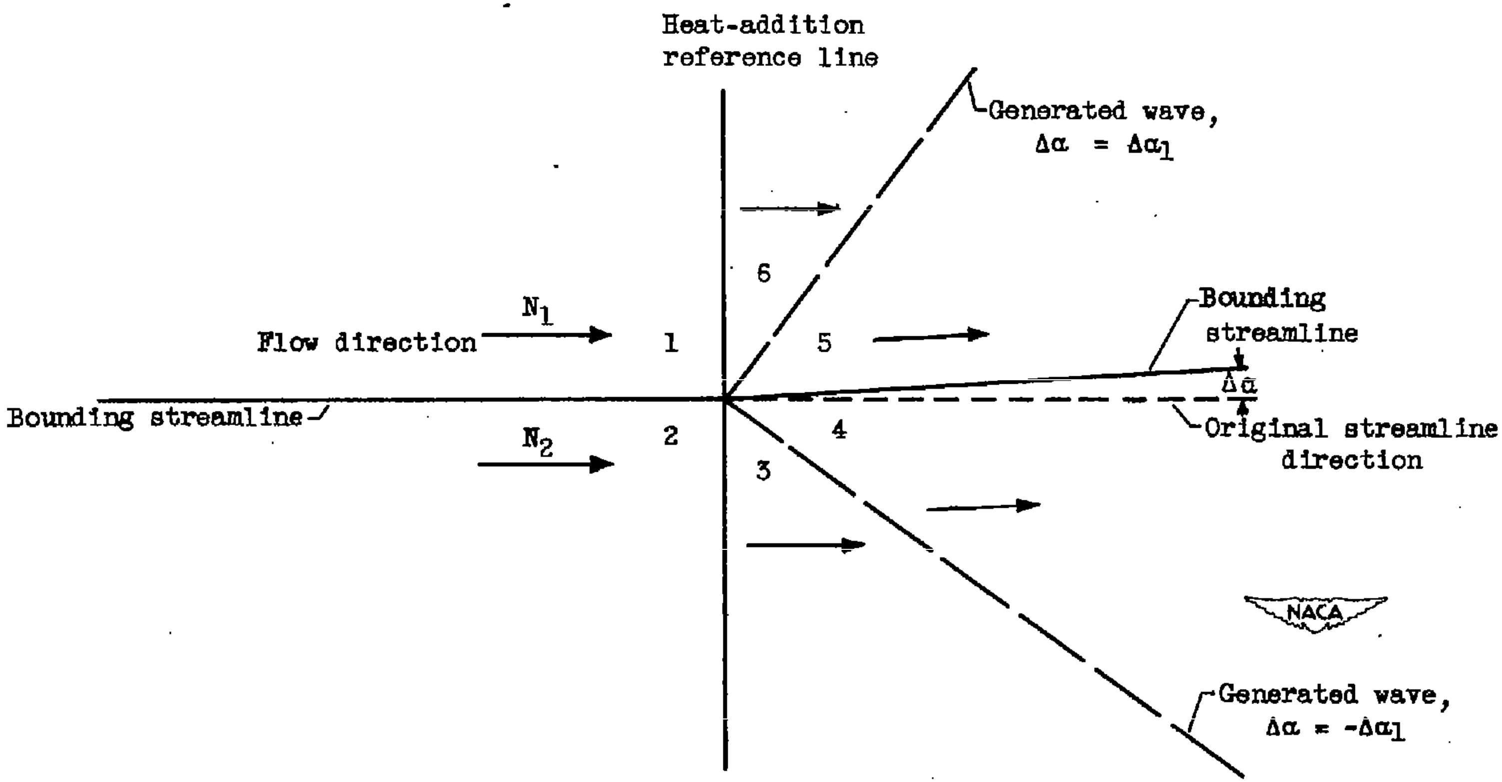


Figure 4. - Schematic representation of wave generation at intersection of heat-addition reference line with adjacent stream tubes of differing total pressure; case IV.

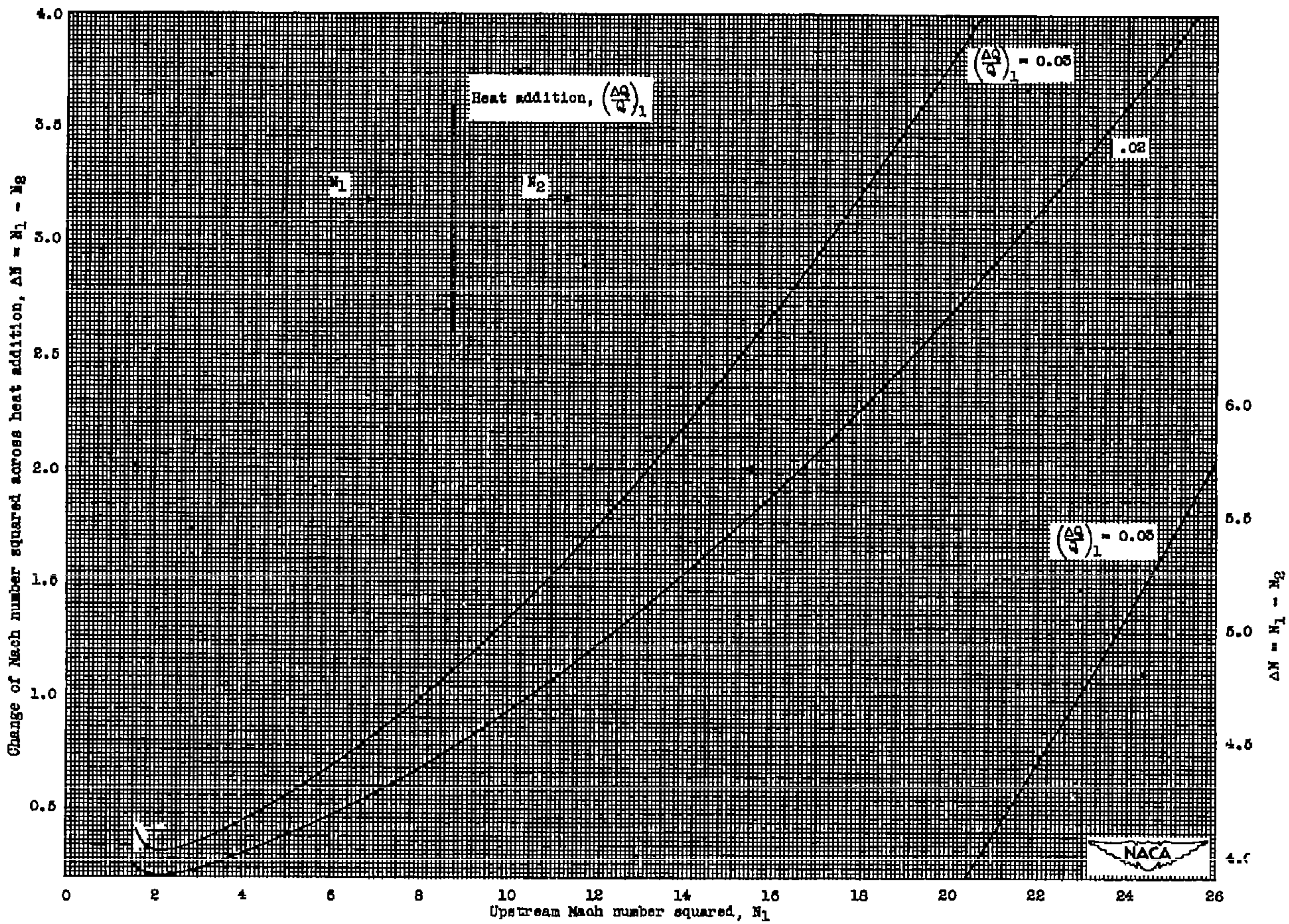


Figure 5. - Variation of change of Mach number squared across heat addition with upstream Mach number squared.

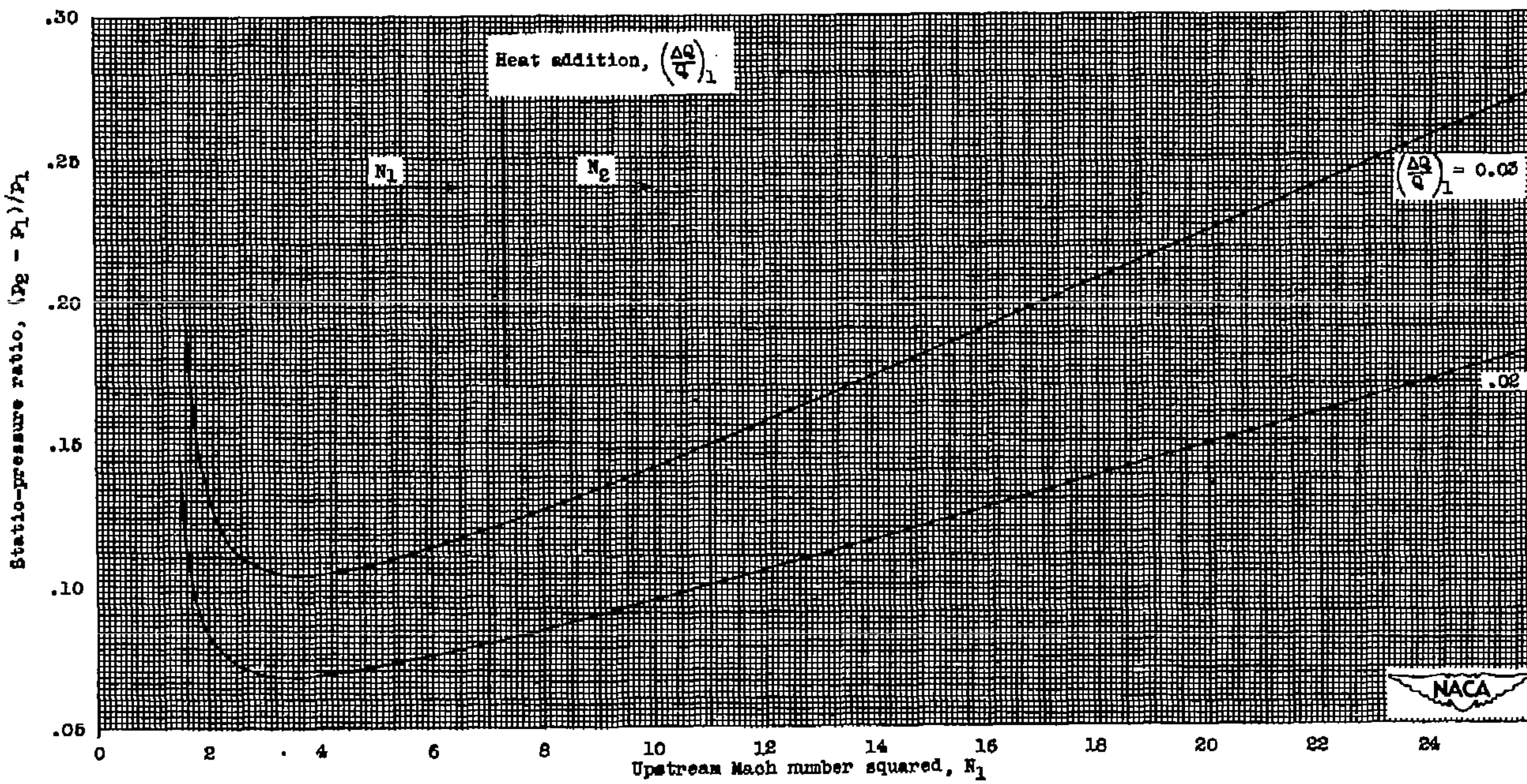


Figure 6. - Variation of ratio of static-pressure change across heat addition to upstream static pressure with upstream Mach number squared.

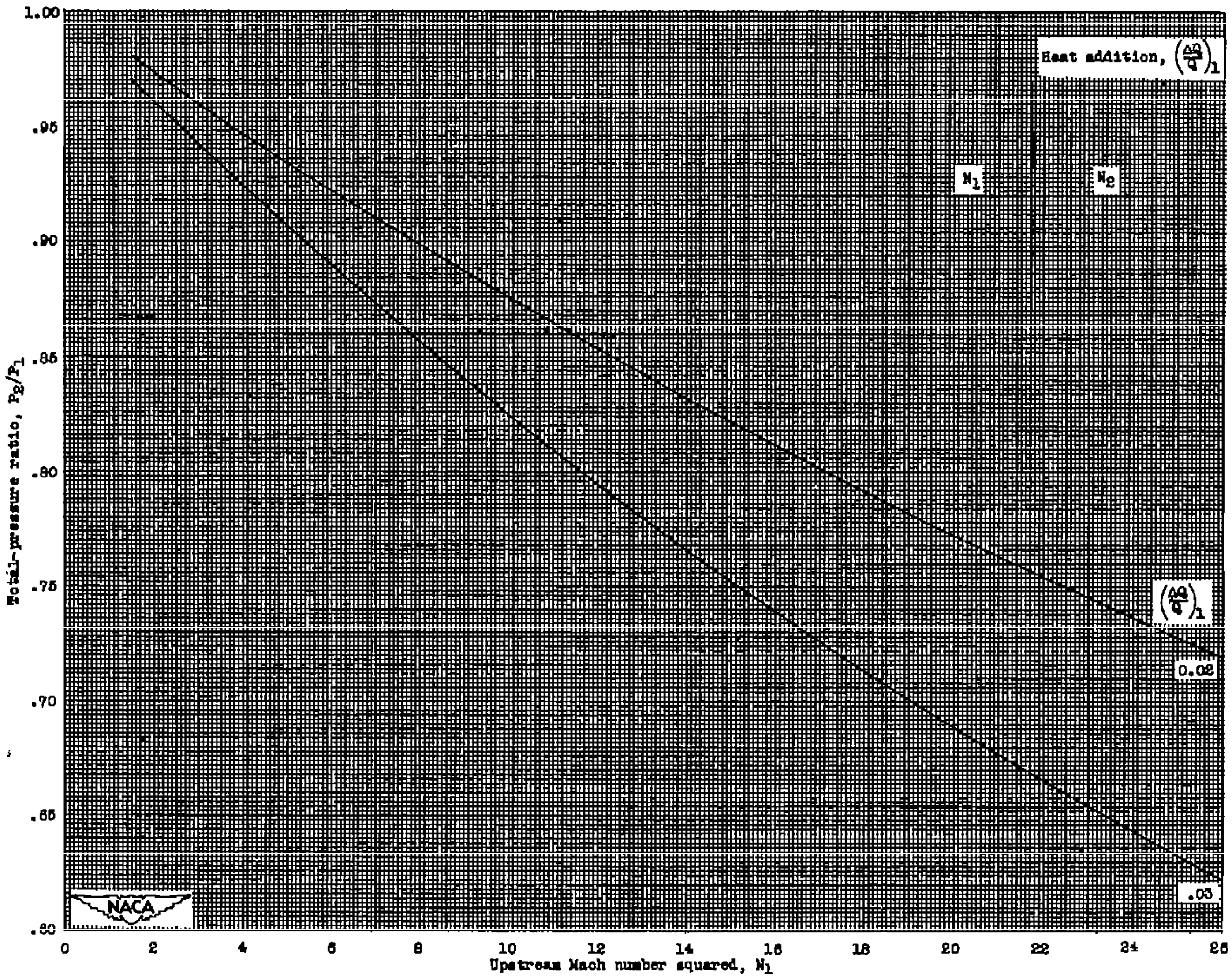
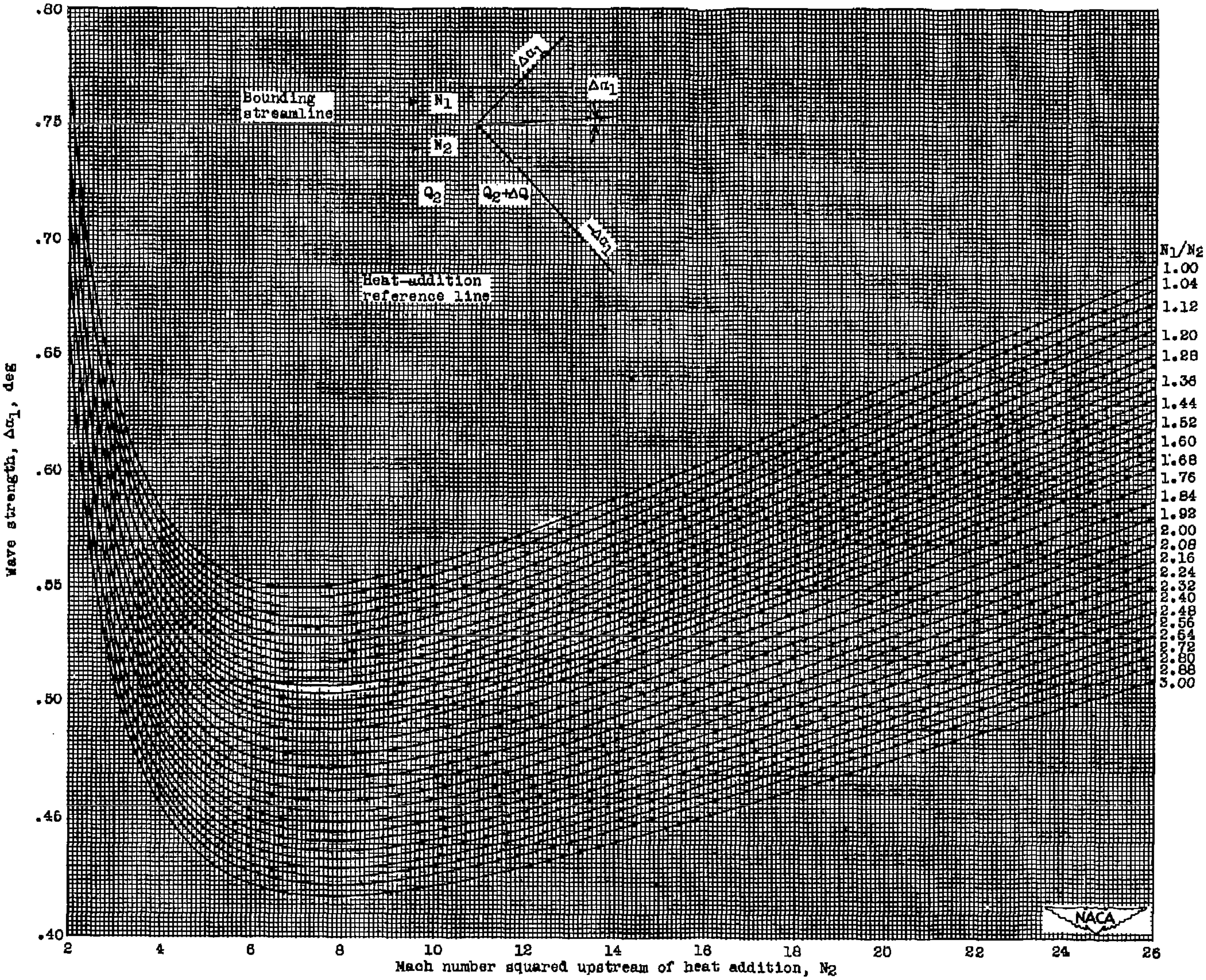
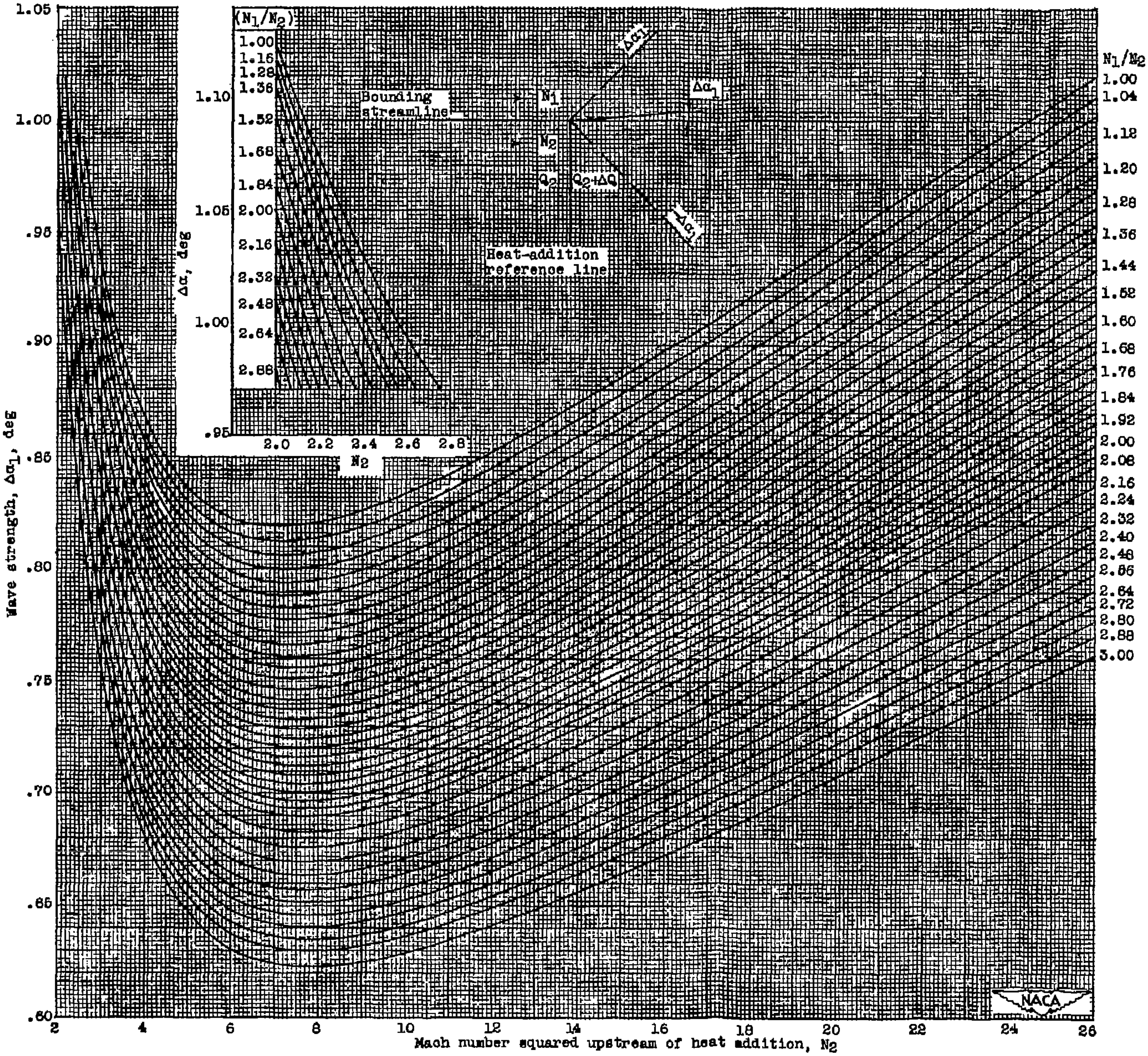


Figure 7. - Variation of ratio of total pressure before heat addition to that after heat addition with upstream Mach number squared.



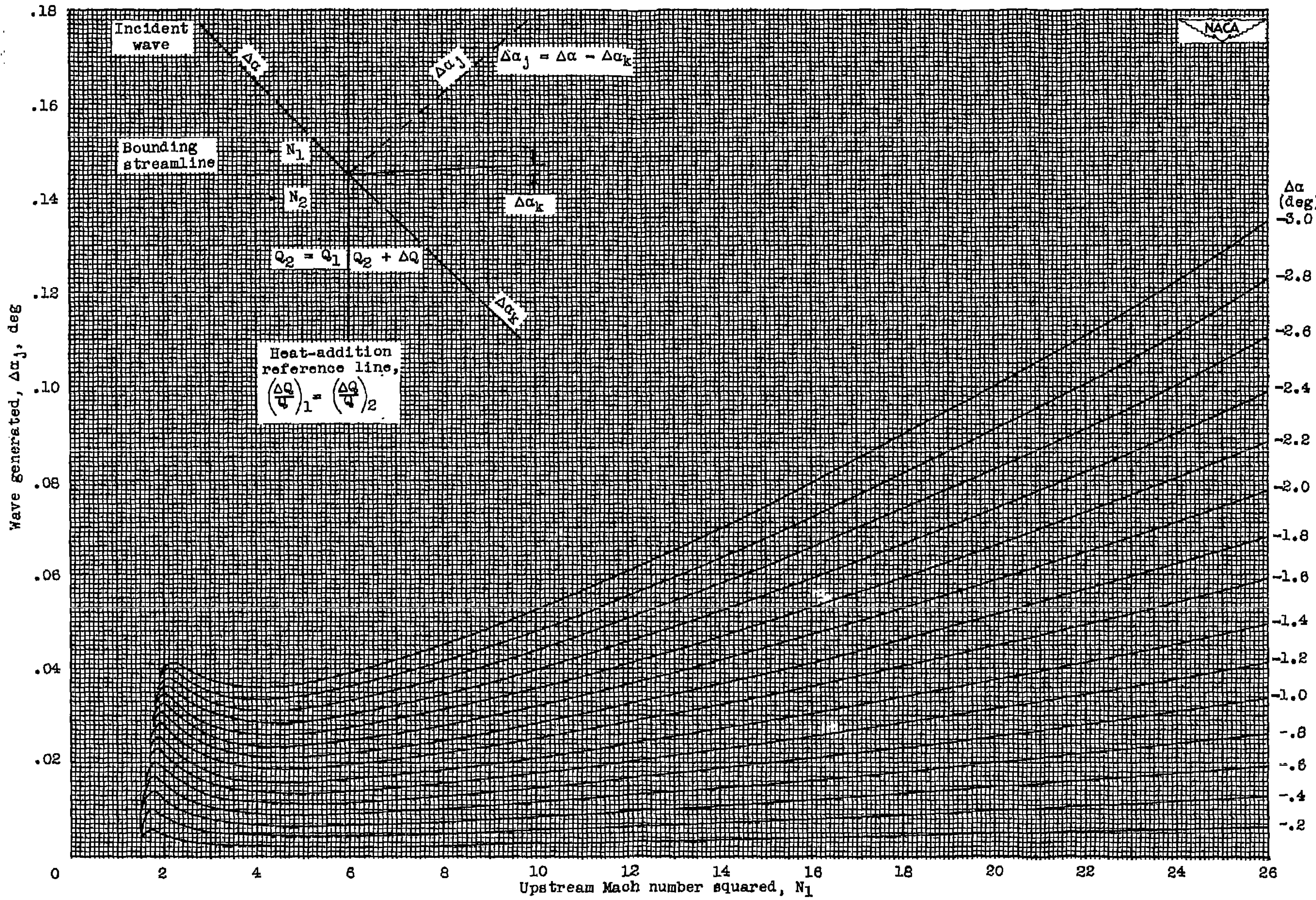
(a) For heat addition, $(\Delta Q/Q)_2 = 0.02$.

Figure 8. - Variation of strength of wave generated at free-stream boundary of heated flow (from equation (12)) with Mach number squared upstream of heat addition; case I.



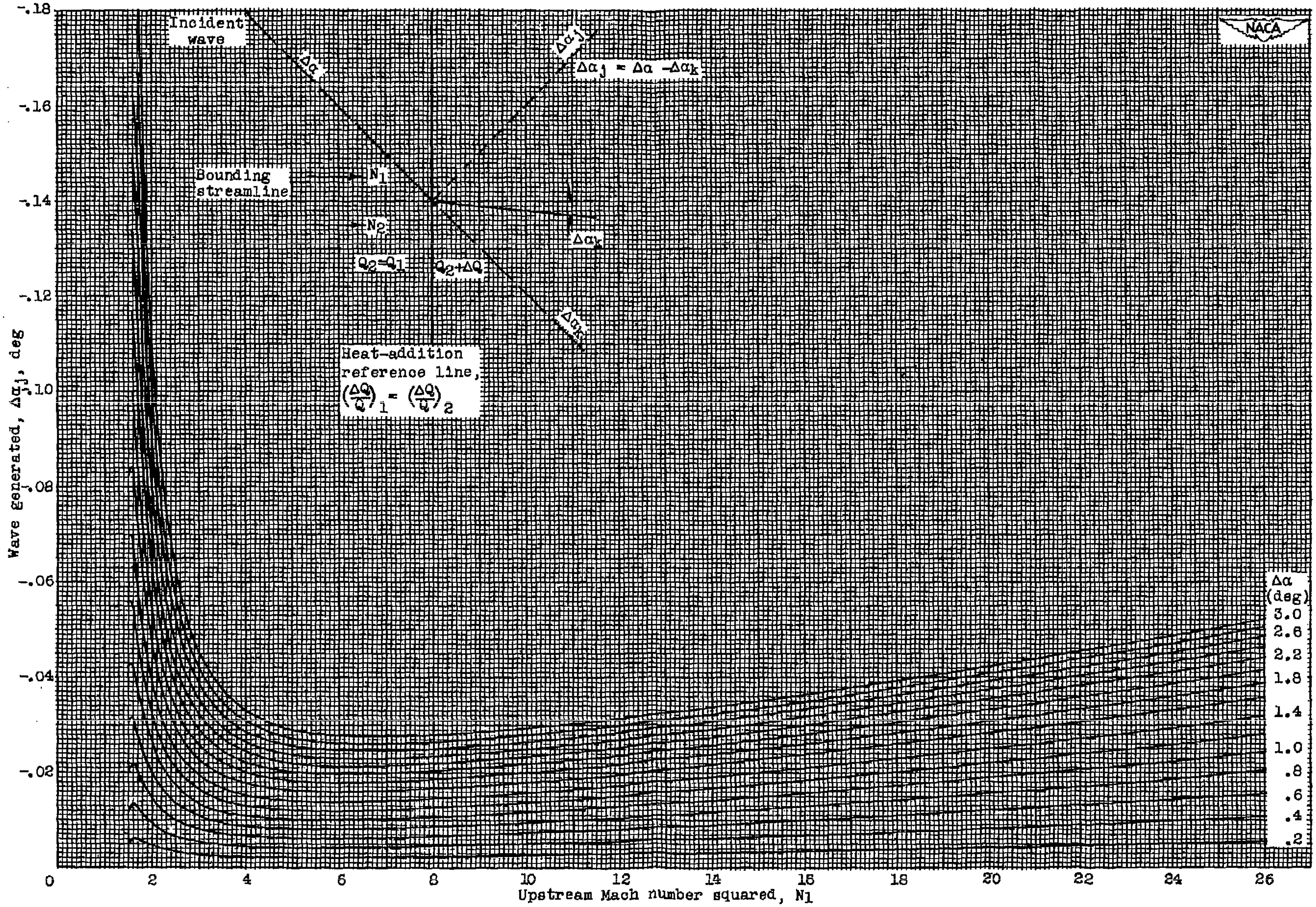
(b) For heat addition, $(\Delta Q/Q)_2 = 0.03$.

Figure 8. - Concluded. Variation of strength of wave generated at free-stream boundary of heated flow (from equation (12)) with Mach number squared upstream of heat addition; case I.



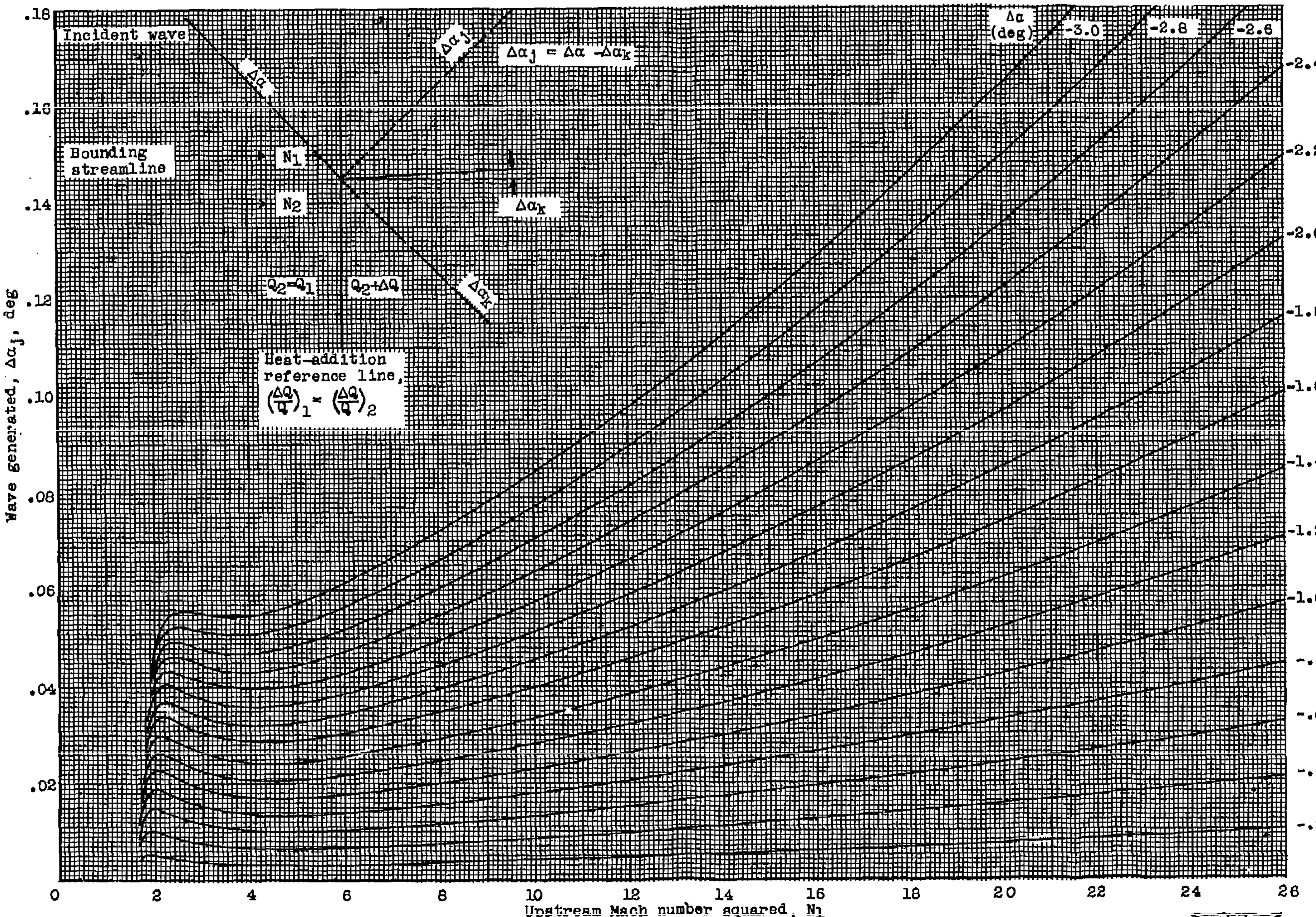
(a) For heat addition $\Delta Q/Q = 0.02$ and negative values of incident-wave strength with $N_1 = N_2$.

Figure 9. - Variation of strength of wave generated at intersection of incident wave and heat-addition reference line with upstream Mach number squared for various incident-wave strengths; case II.



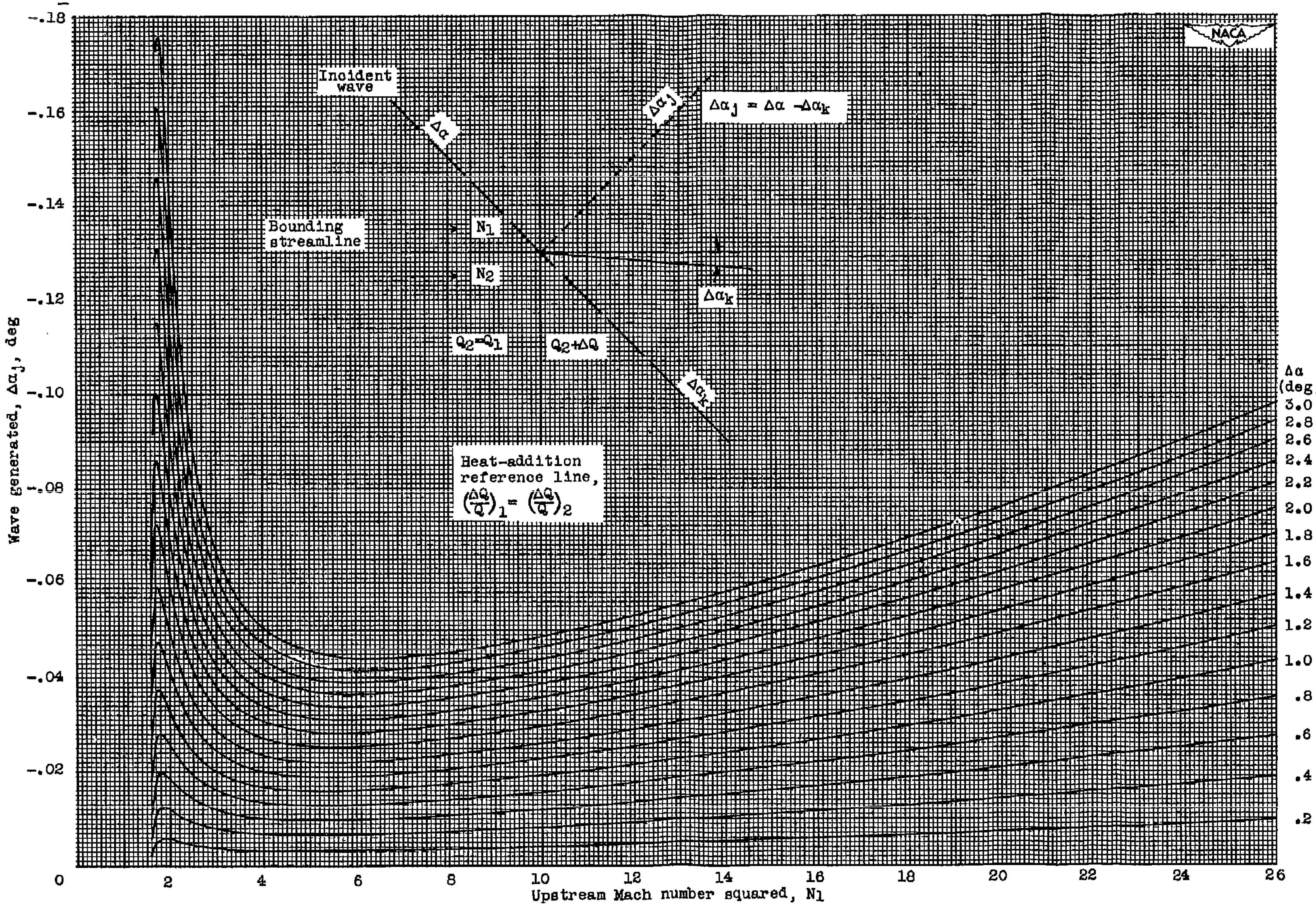
(b) For heat addition $\Delta Q/Q = 0.02$ and positive values of incident-wave strength with $N_1 = N_2$.

Figure 9. - Continued. Variation of strength of wave generated at intersection of incident wave and heat-addition reference line with upstream Mach number squared for various incident-wave strengths; case II.



(c) For heat addition of $\Delta Q/Q = 0.03$ and negative values of incident-wave strength with $N_1 = N_2$.

Figure 9. - Continued. Variation of strength of wave generated at intersection of incident wave and heat-addition reference line with upstream Mach number squared for various incident-wave strengths; case II.



(d) For heat addition $\Delta Q/Q = 0.03$ and positive values of incident-wave strength with $N_1 = N_2$.

Figure 9. - Concluded. Variation of strength of wave generated at intersection of incident wave and heat-addition reference line with upstream Mach number squared for various incident-wave strengths; case II.

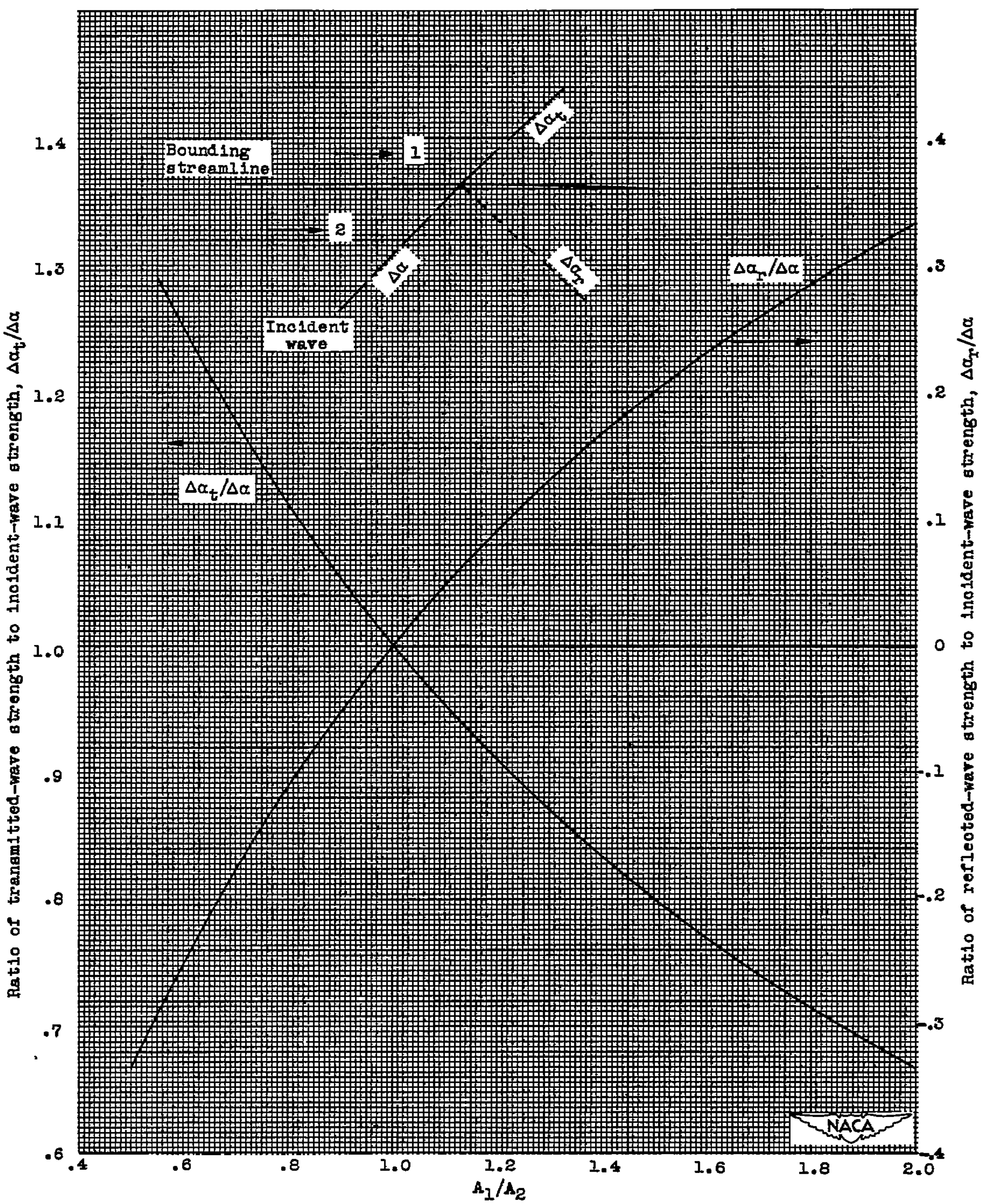


Figure 10. - Variation of ratio of transmitted-wave strength to incident-wave strength and ratio of reflected-wave strength to incident-wave strength with ratio A_1/A_2, where $A = \gamma N/\sqrt{N-1}$; case III, equation (23b).

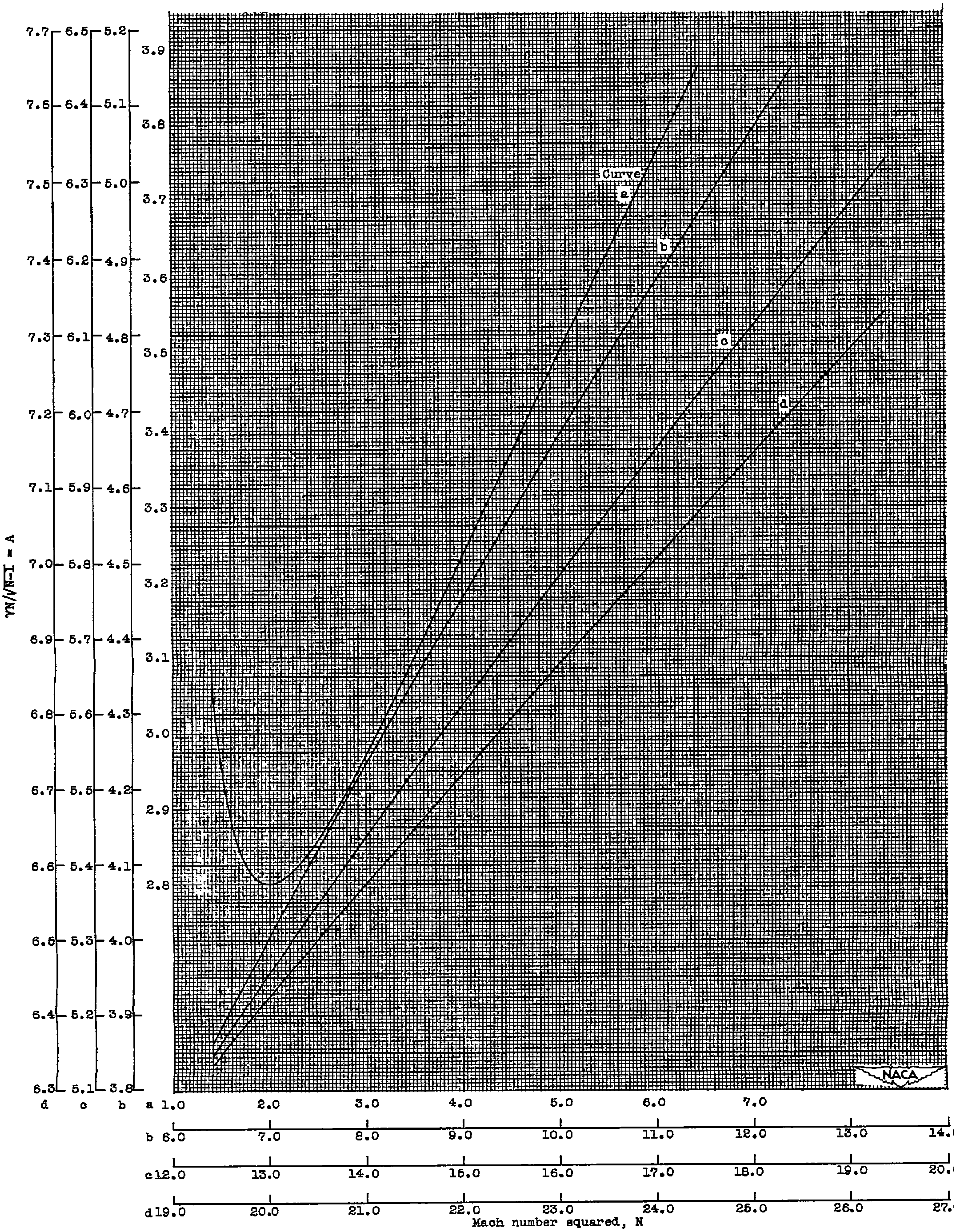
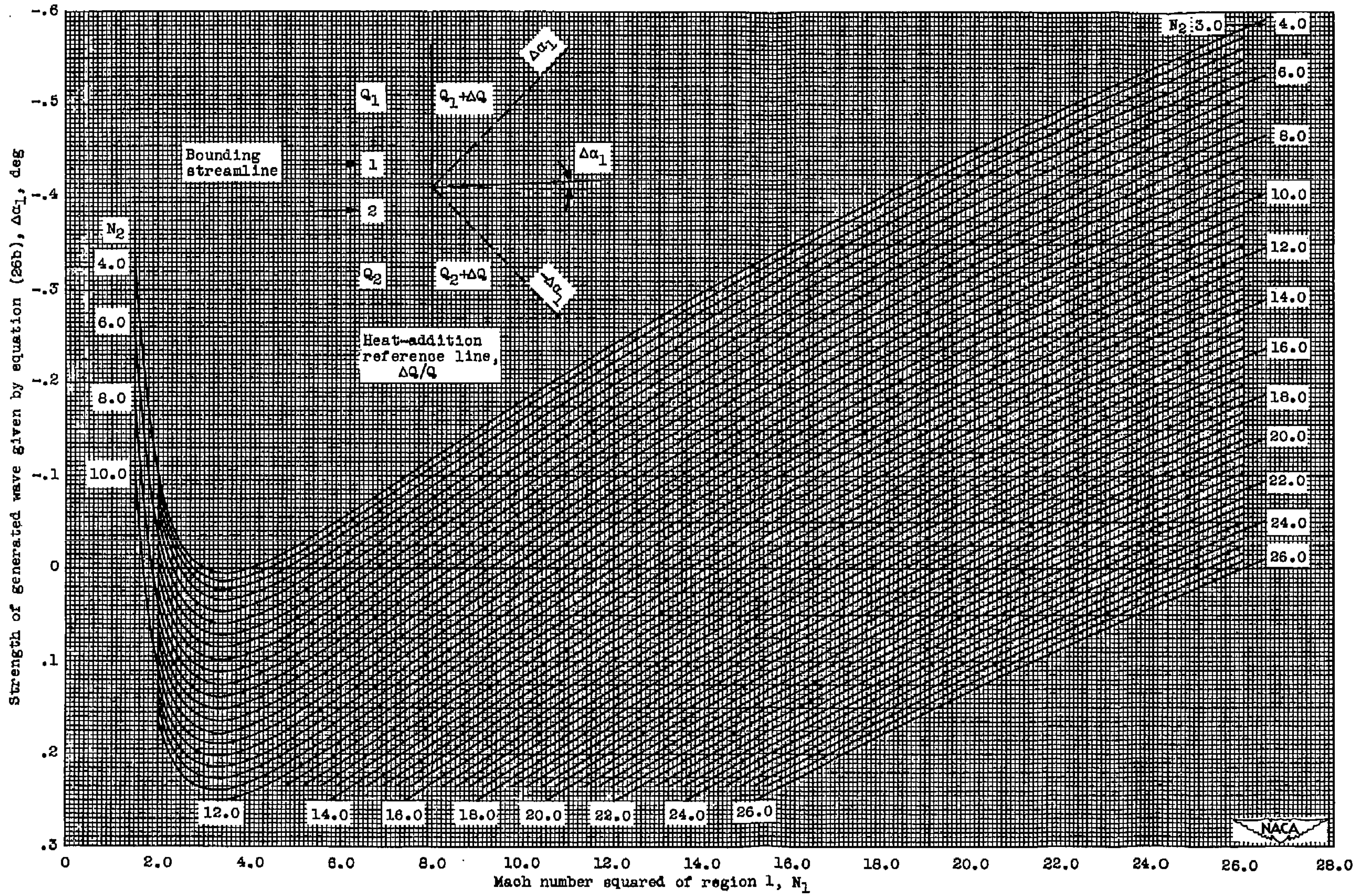
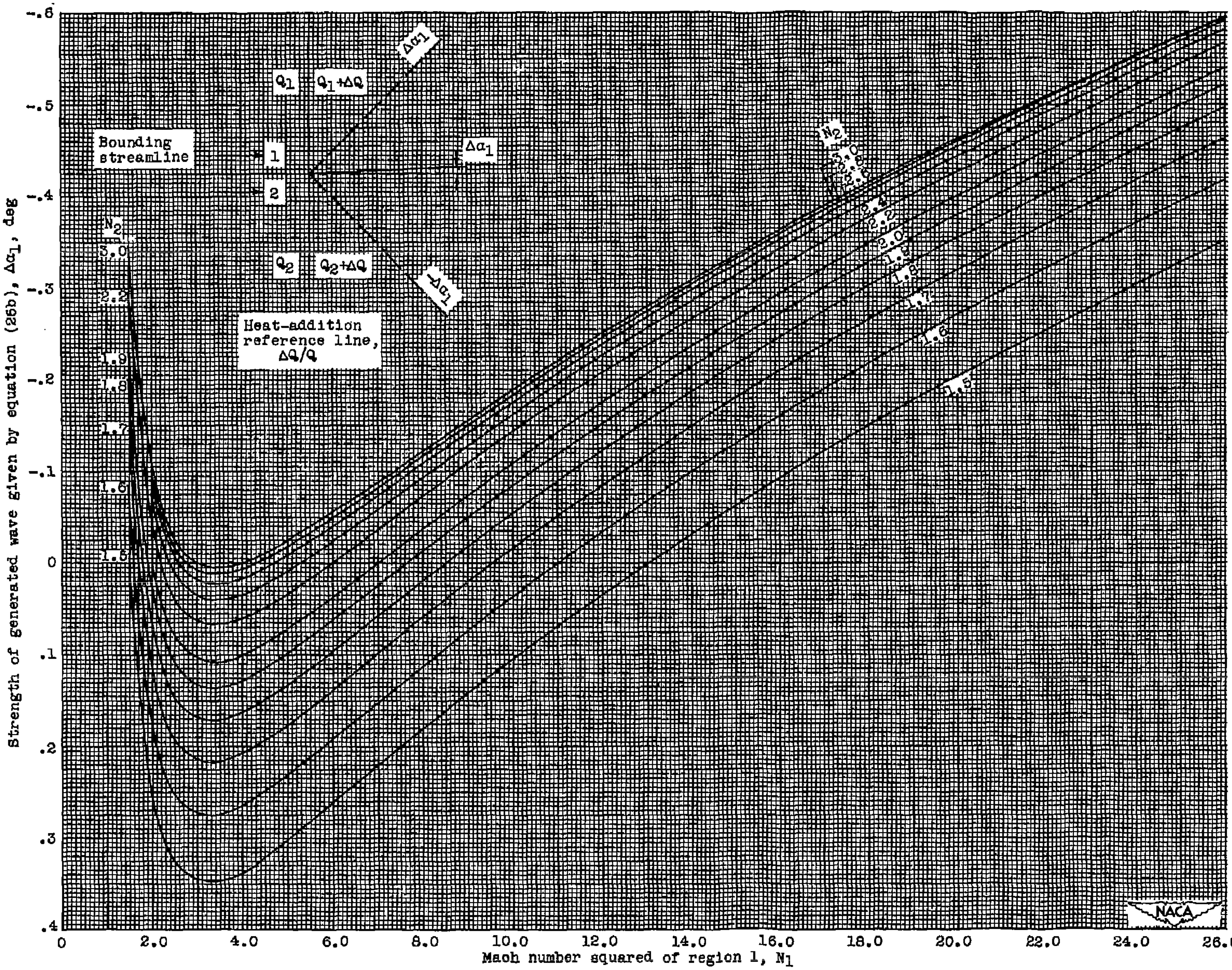


Figure 11. - Variation of γN/√N-1 with Mach number squared.



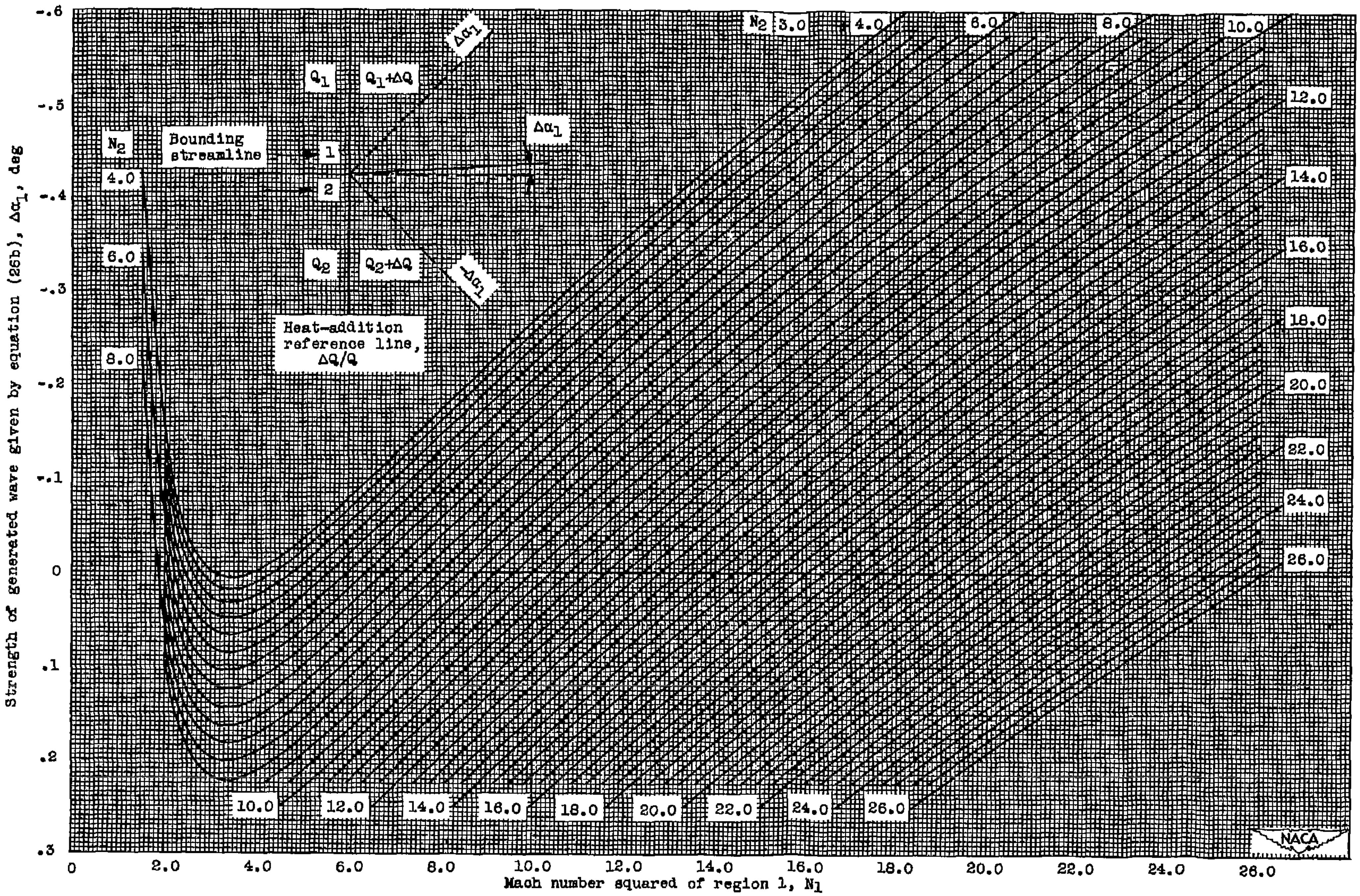
(a) For heat addition $\Delta Q/Q = 0.02$ and values of $N_2 \geq 3.0$.

Figure 12. - Variation of strength of wave generated by heat addition across stream tubes of differing total pressures with Mach number squared of region 1 for various values of Mach number squared of region 2; case IV.



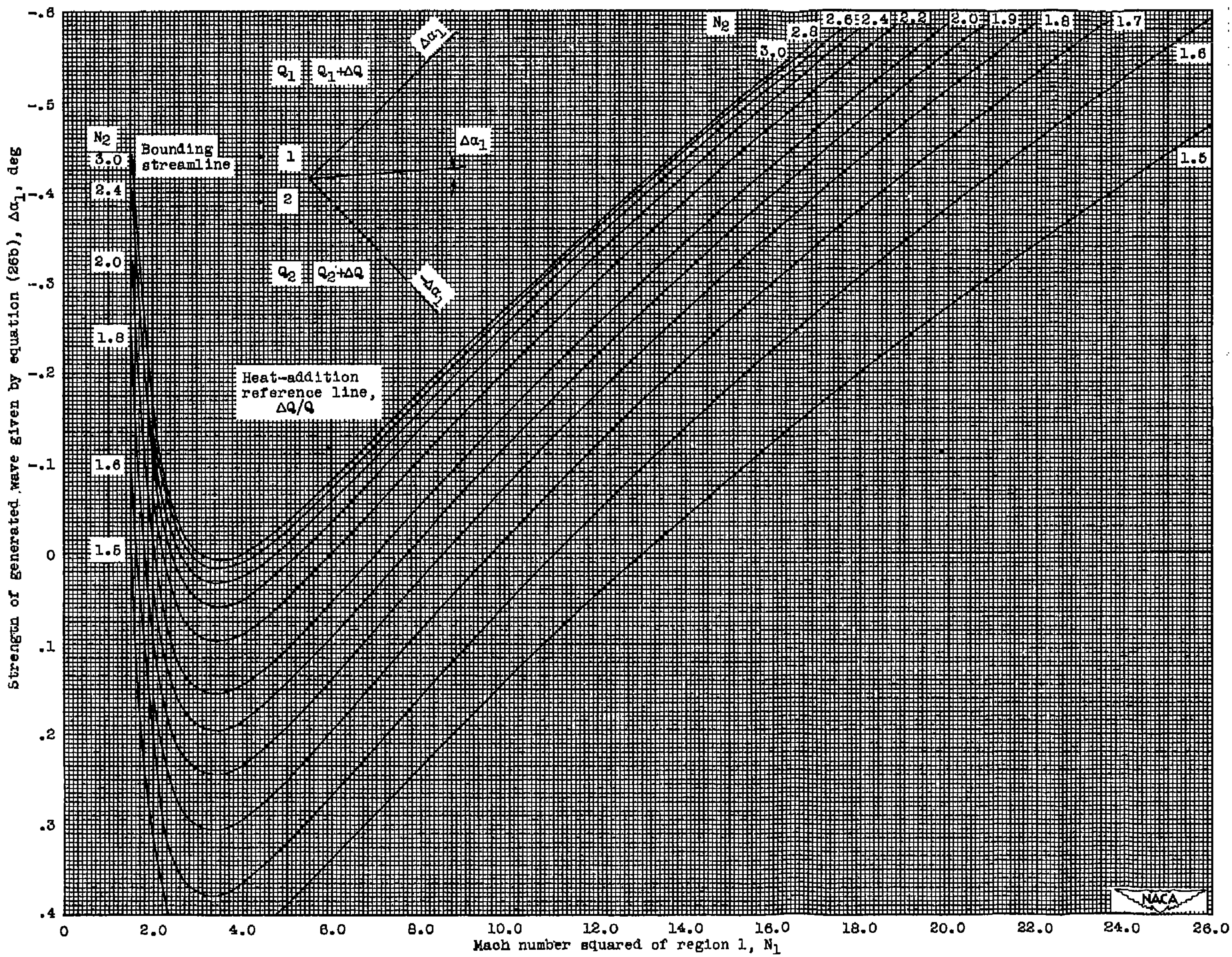
(b) For heat addition $\Delta Q/Q = 0.02$ and values of $N_2 \leqq 3.0$.

Figure 12. - Continued. Variation of strength of wave generated by heat addition across stream tubes of differing total pressures with Mach number squared of region 1 for various values of Mach number squared of region 2; case IV.



(c) For heat addition of 0.03 and values of $N_2 \geq 3.0$.

Figure 12. - Continued. Variation of strength of wave generated by heat addition across stream tubes of differing total pressures with Mach number squared of region 1 for various values of Mach number squared of region 2; case IV.



(d) For heat addition $\Delta Q/Q = 0.03$ and values of $N_2 \leq 3.0$.

Figure 12. - Concluded. Variation of strength of wave generated by heat addition across stream tubes of differing total pressures with Mach number squared of region 1 for various values of Mach number squared of region 2; case IV.

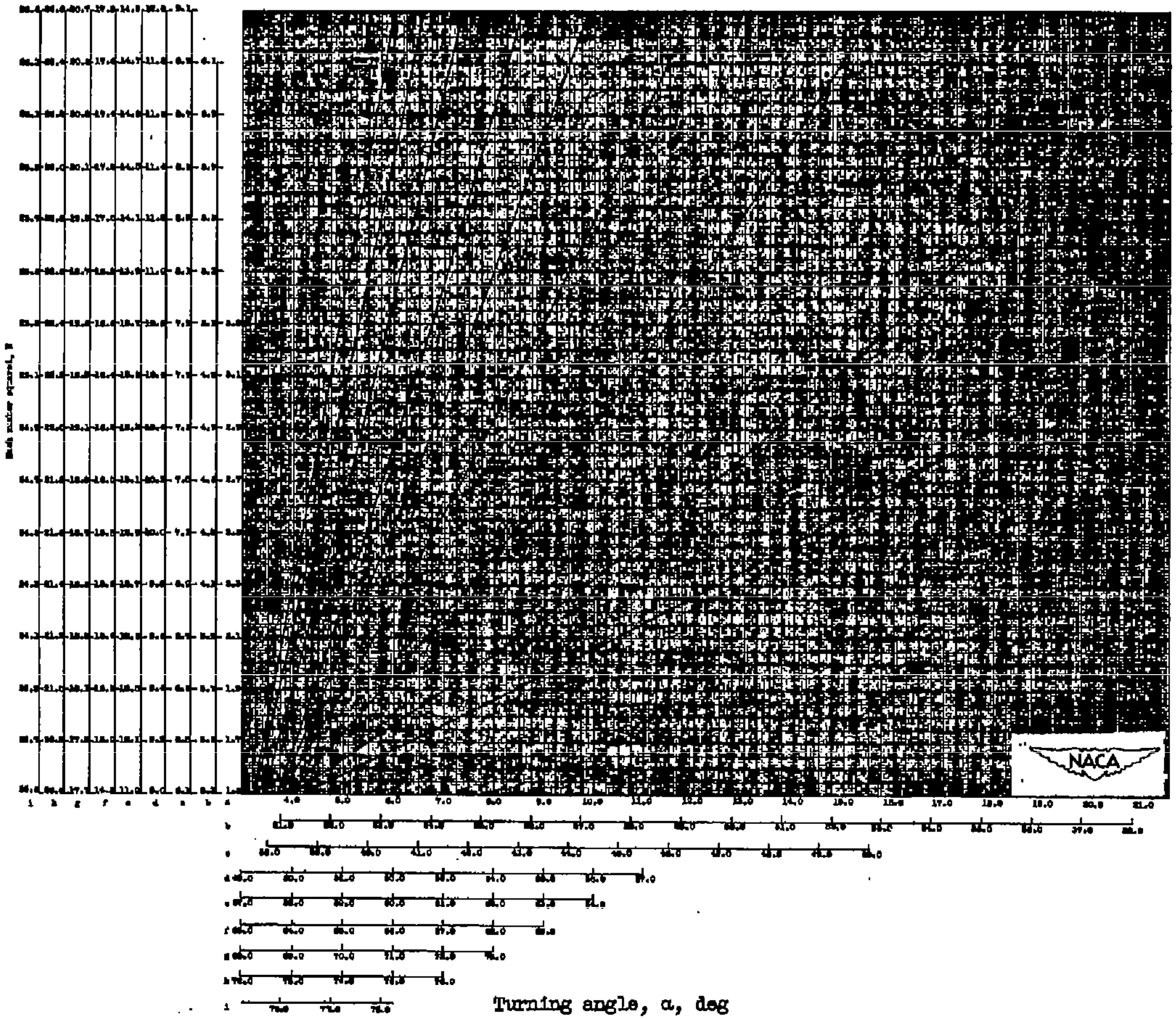


Figure 13. - Variation of Mach number squared with angle through which supersonic stream is turned to expand from $M = 1$ to $M > 1$. (A 17- by 22-in. print of this fig. is attached.)

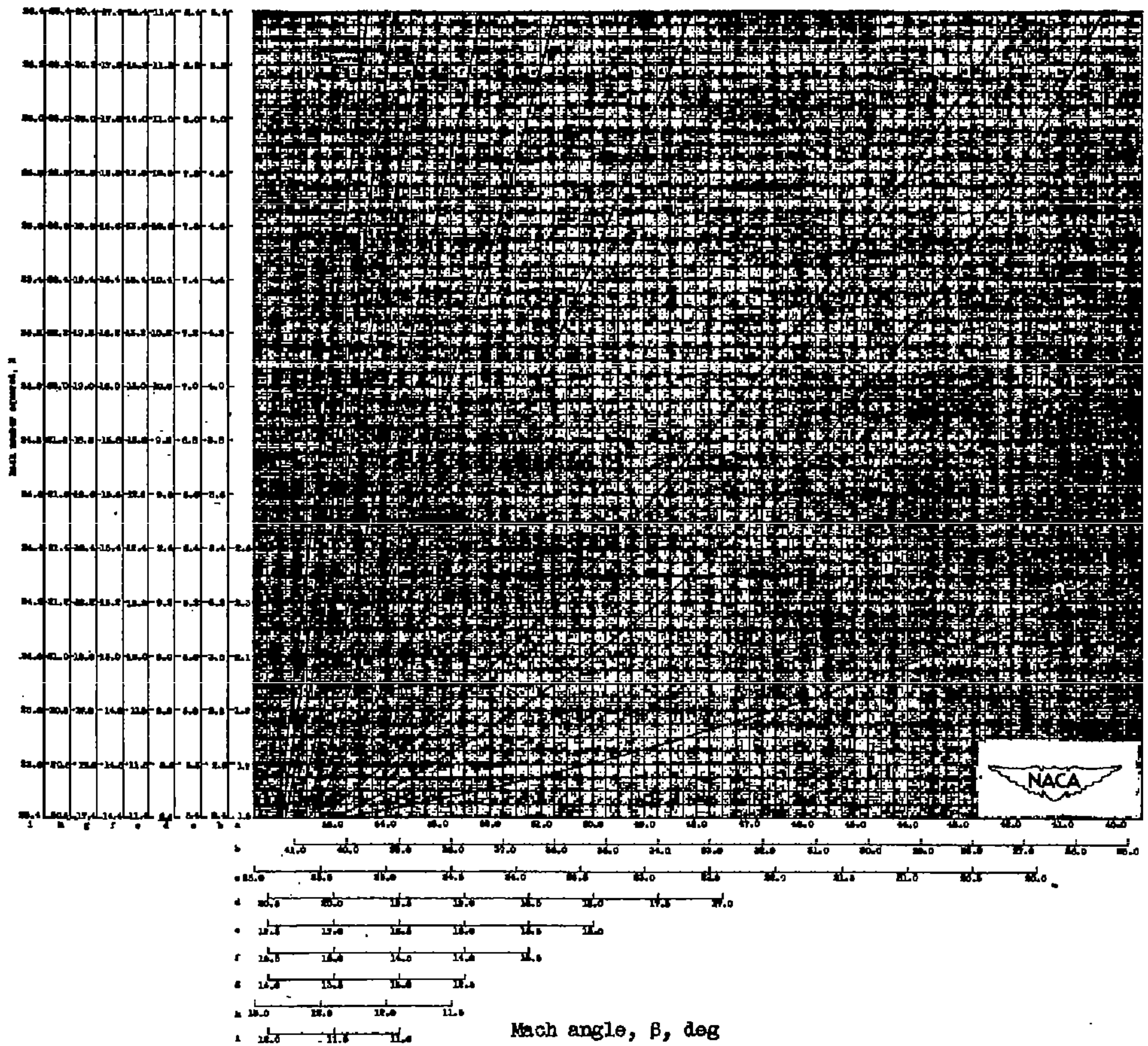


Figure 14. - Variation of Mach number squared with Mach angle. (A 17- by 22-in. print of this fig. is attached.)

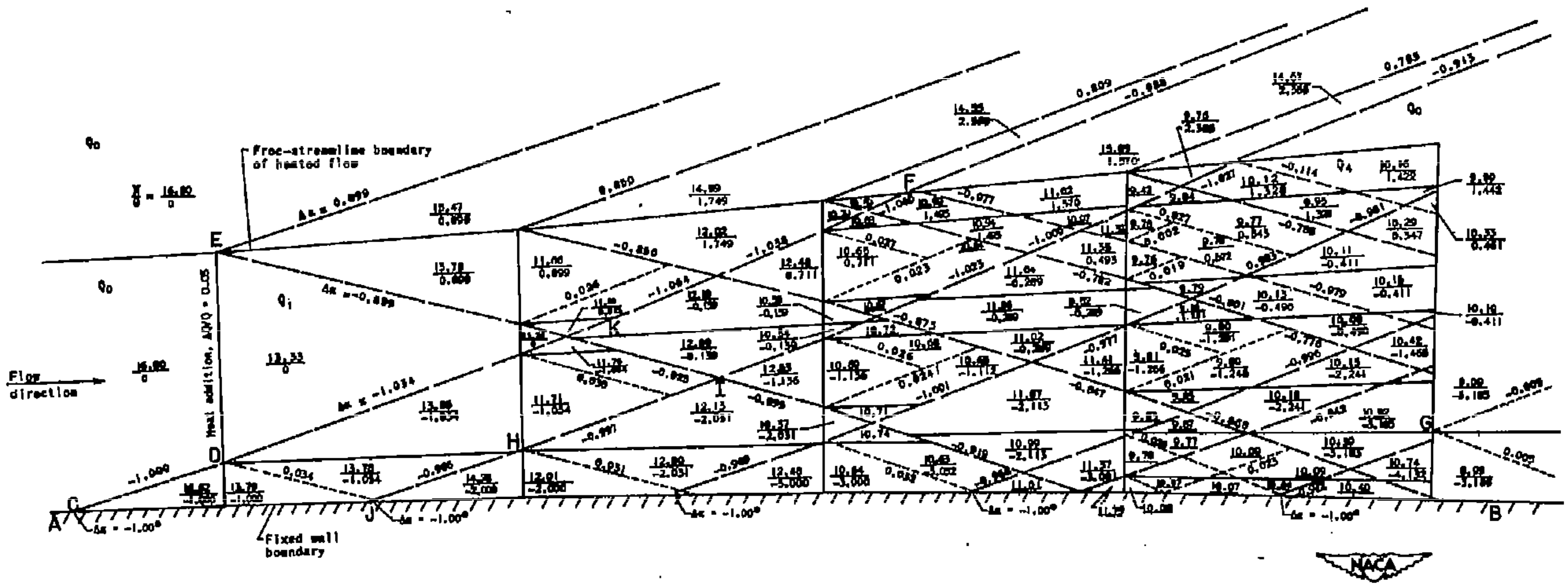
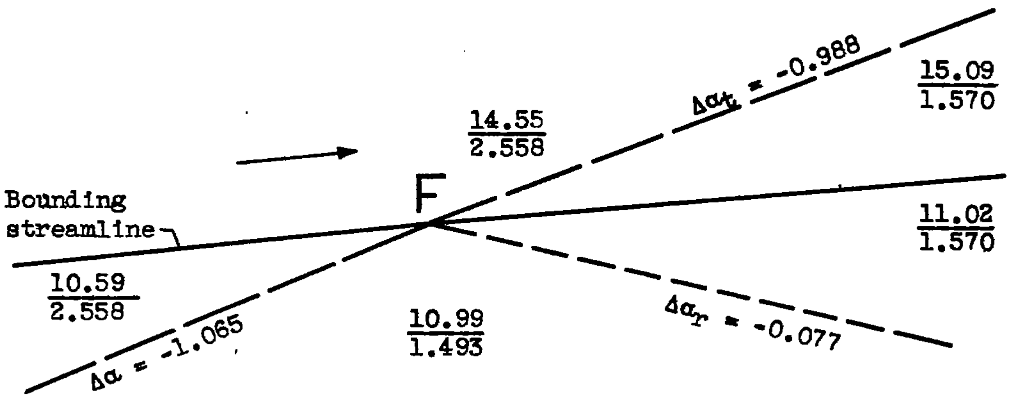
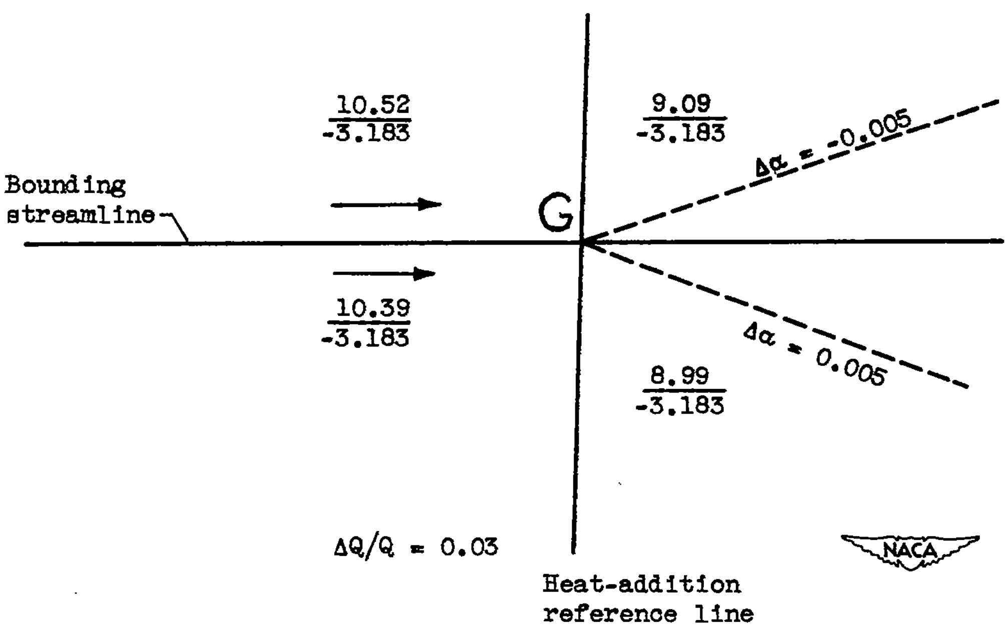


Figure 15. - Example showing use of method in obtaining graphical solutions for two-dimensional supersonic flows with heat addition.

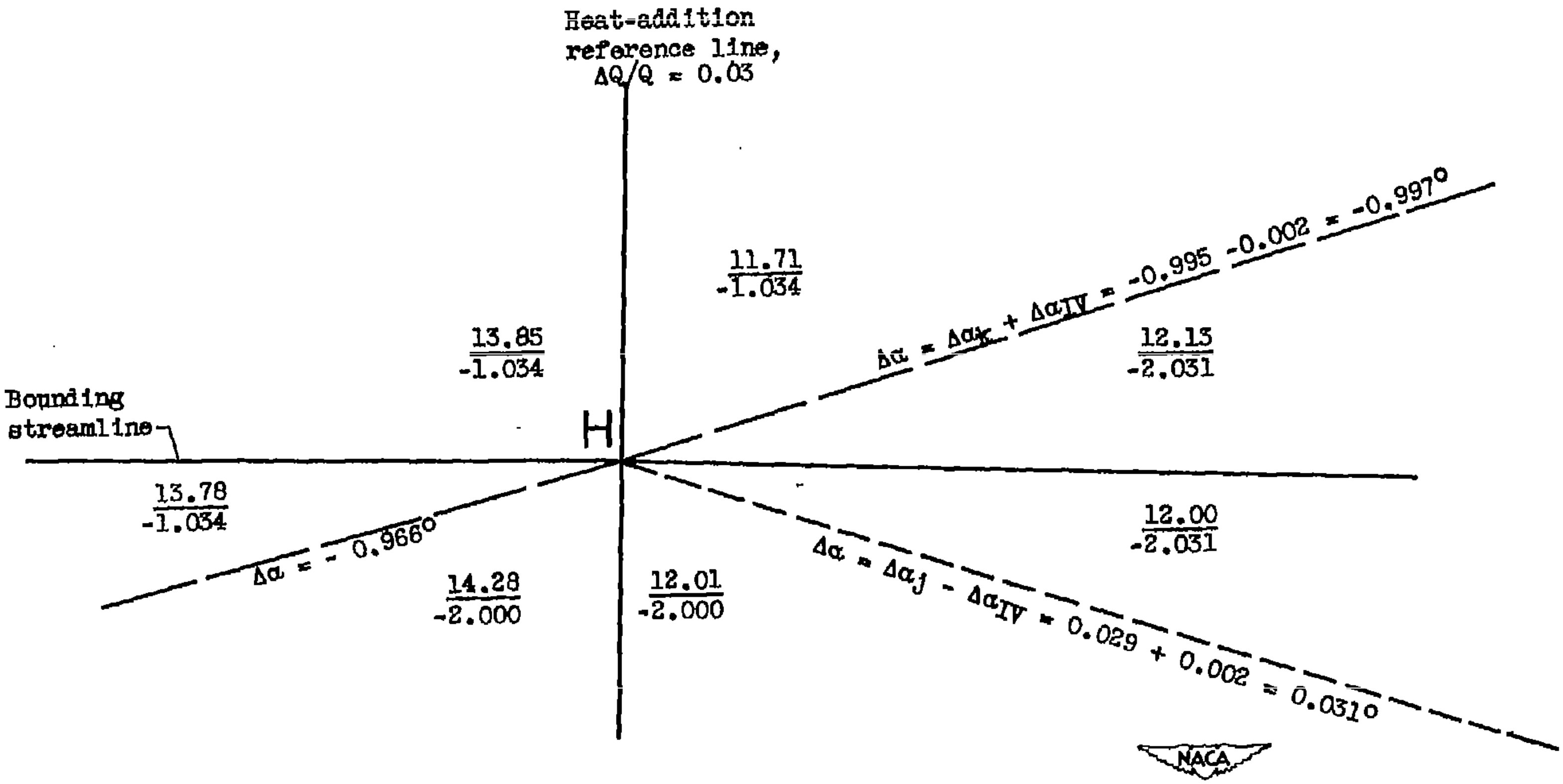


(a) Section about point F, showing wave crossing adjacent stream tubes of differing total pressure; case III.



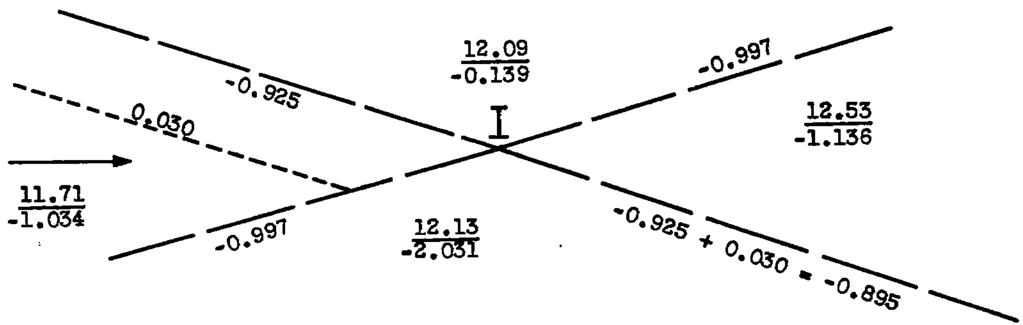
(b) Section about point G, showing addition of heat across adjacent stream tubes of differing total pressure; case IV.

Figure 16. - Enlarged sections of graphical example shown in figure 15, illustrating in detail several cases.

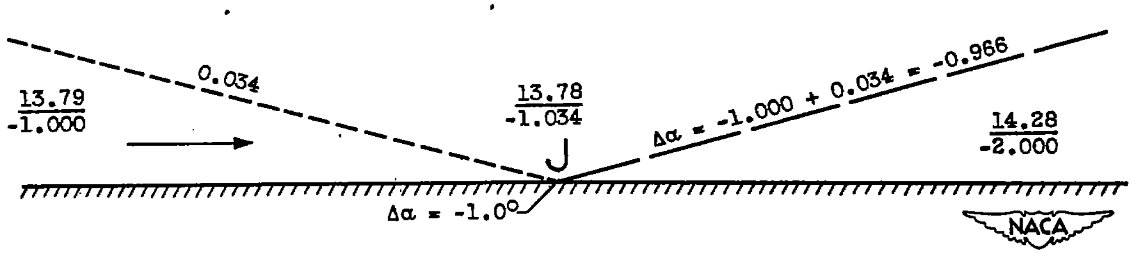


(c) Section about point H showing intersection of wave and streamline separating flows of differing total pressure with heat-addition reference line; cases II and IV.

Figure 16. - Concluded. Enlarged sections of graphical example shown in figure 15, illustrating in detail several cases.

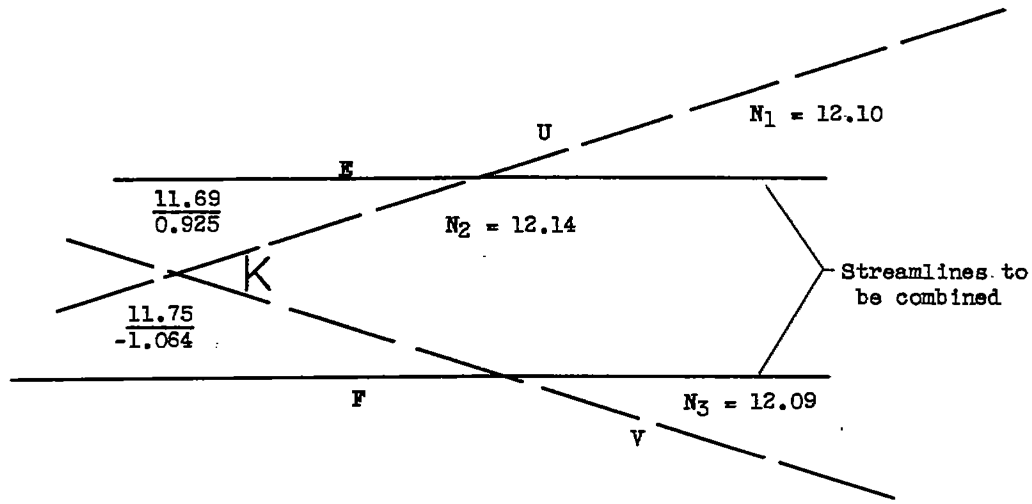


(a) Section about point I showing method of reducing number of waves in flow (combination of minor wave with strong wave).

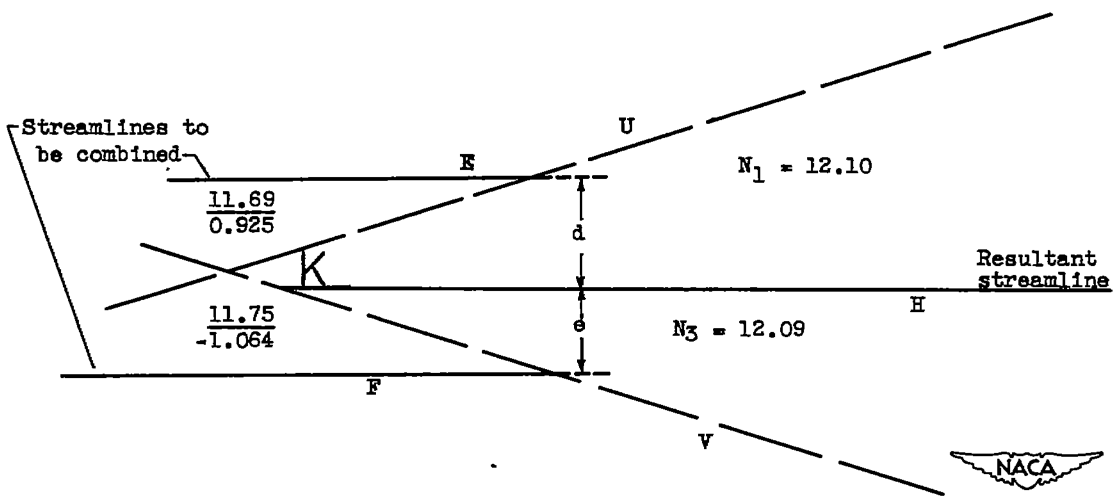


(b) Section about point J showing method of reducing number of waves in flow (combination of minor wave with strong wave caused by deflection of wall boundary).

Figure 17. - Enlarged sections of graphical example shown in figure 15, presenting methods for reducing complexity of flow pattern.



(c) Section about point K before combination of streamlines E and F into a single streamline.



(d) Section about point K after combination of streamlines E and F into resultant streamline H.

Figure 17. - Concluded. Enlarged sections of graphical example shown in figure 15, presenting methods for reducing complexity of flow pattern.

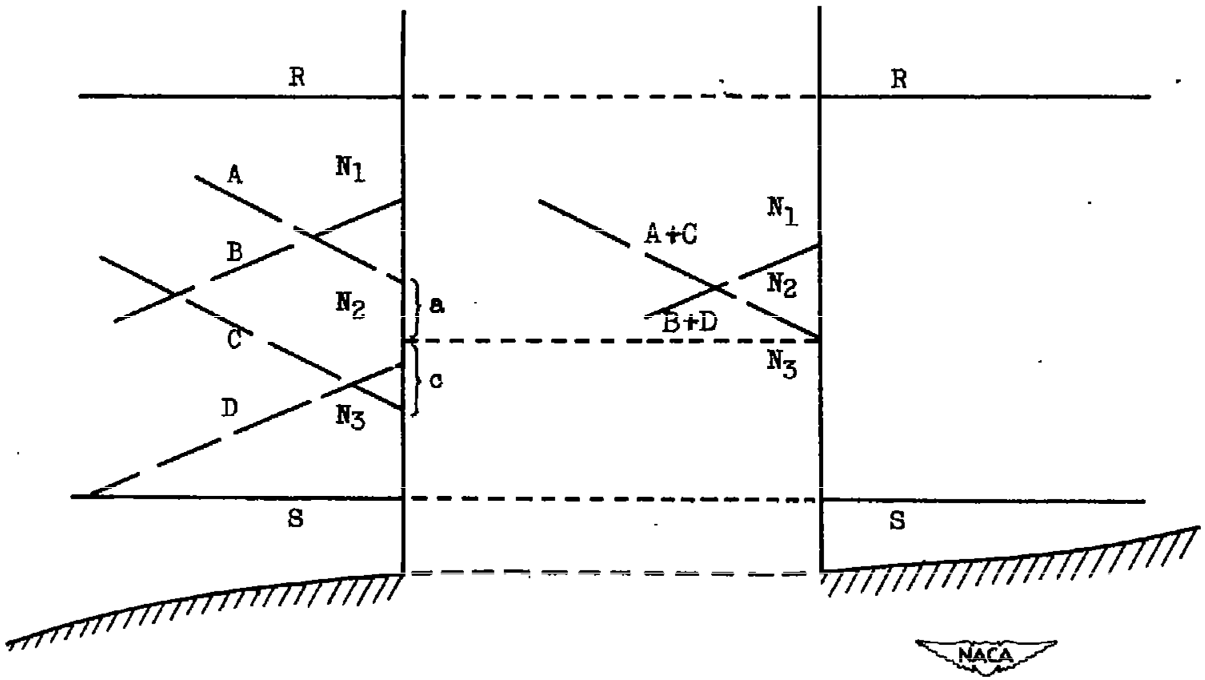


Figure 18. - Sketch of method of reducing number of waves.

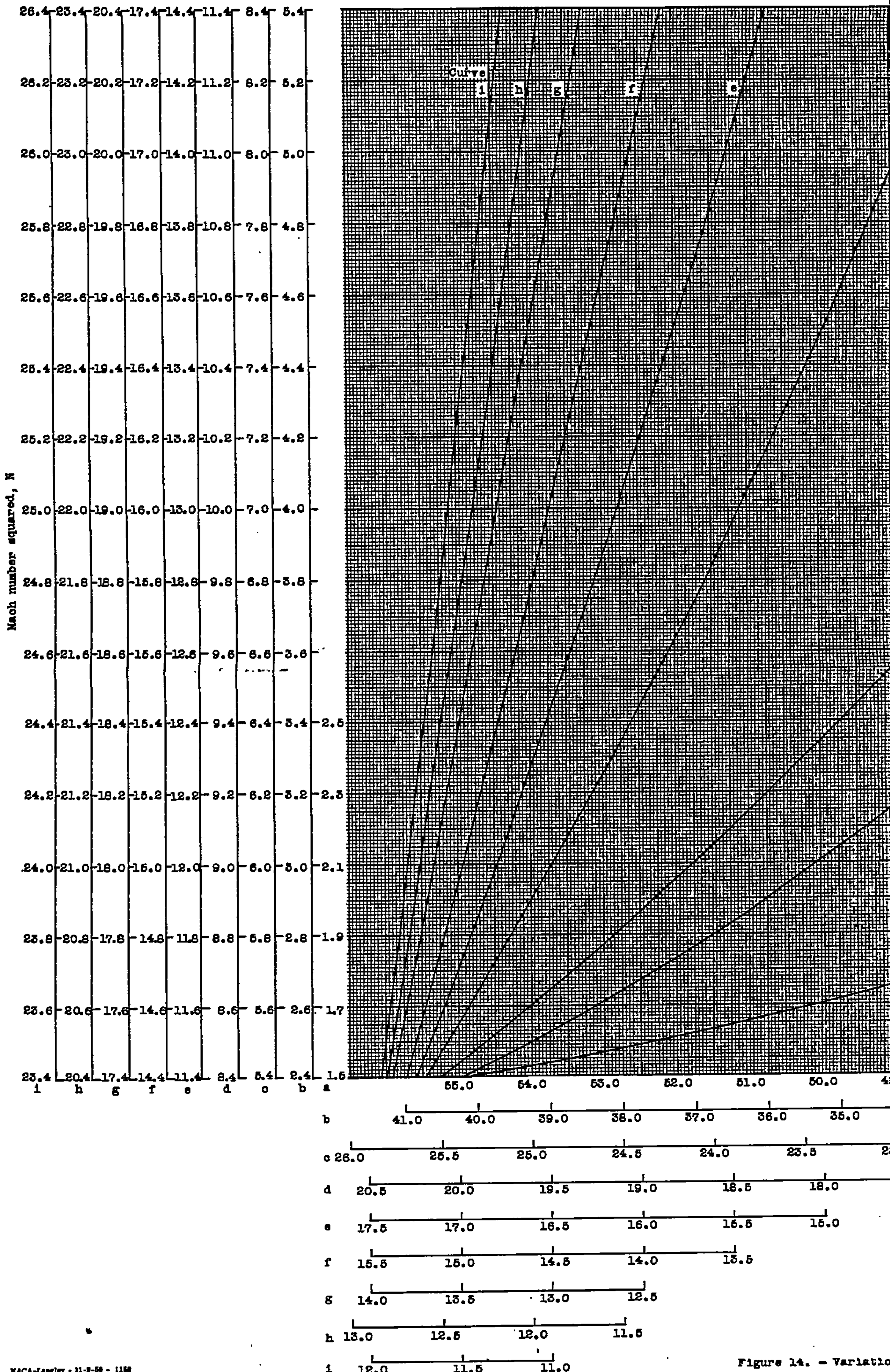
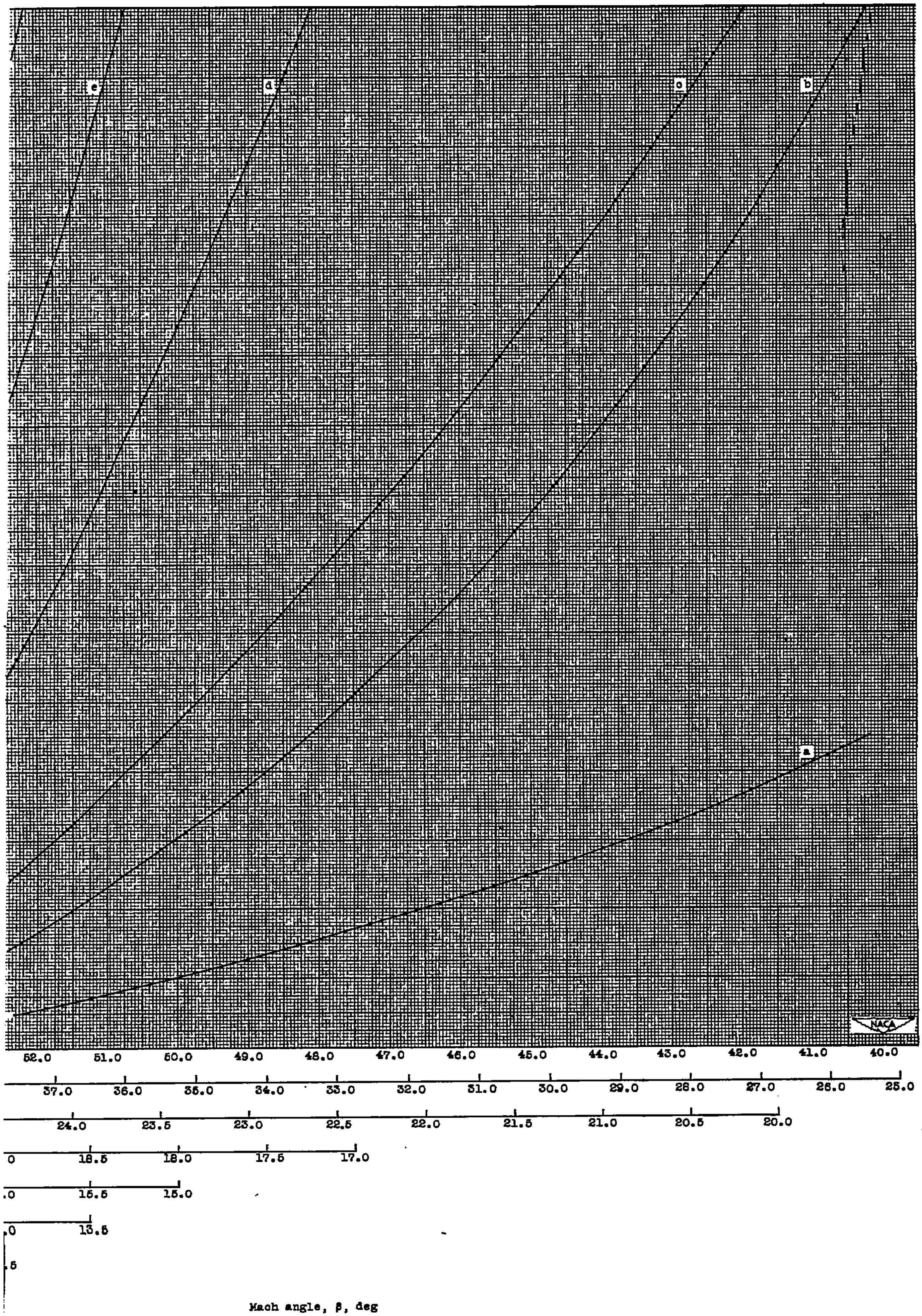


Figure 14. – Variatio



e
d
c
b
a
NACA
52.0 51.0 50.0 49.0 48.0 47.0 46.0 45.0 44.0 43.0 42.0 41.0 40.0
37.0 36.0 35.0 34.0 33.0 32.0 31.0 30.0 29.0 28.0 27.0 26.0 25.0
24.0 23.5 23.0 22.5 22.0 21.5 21.0 20.5 20.0
0 18.5 18.0 17.5 17.0
.0 15.5 15.0
.0 13.5
.5
Mach angle, β, deg

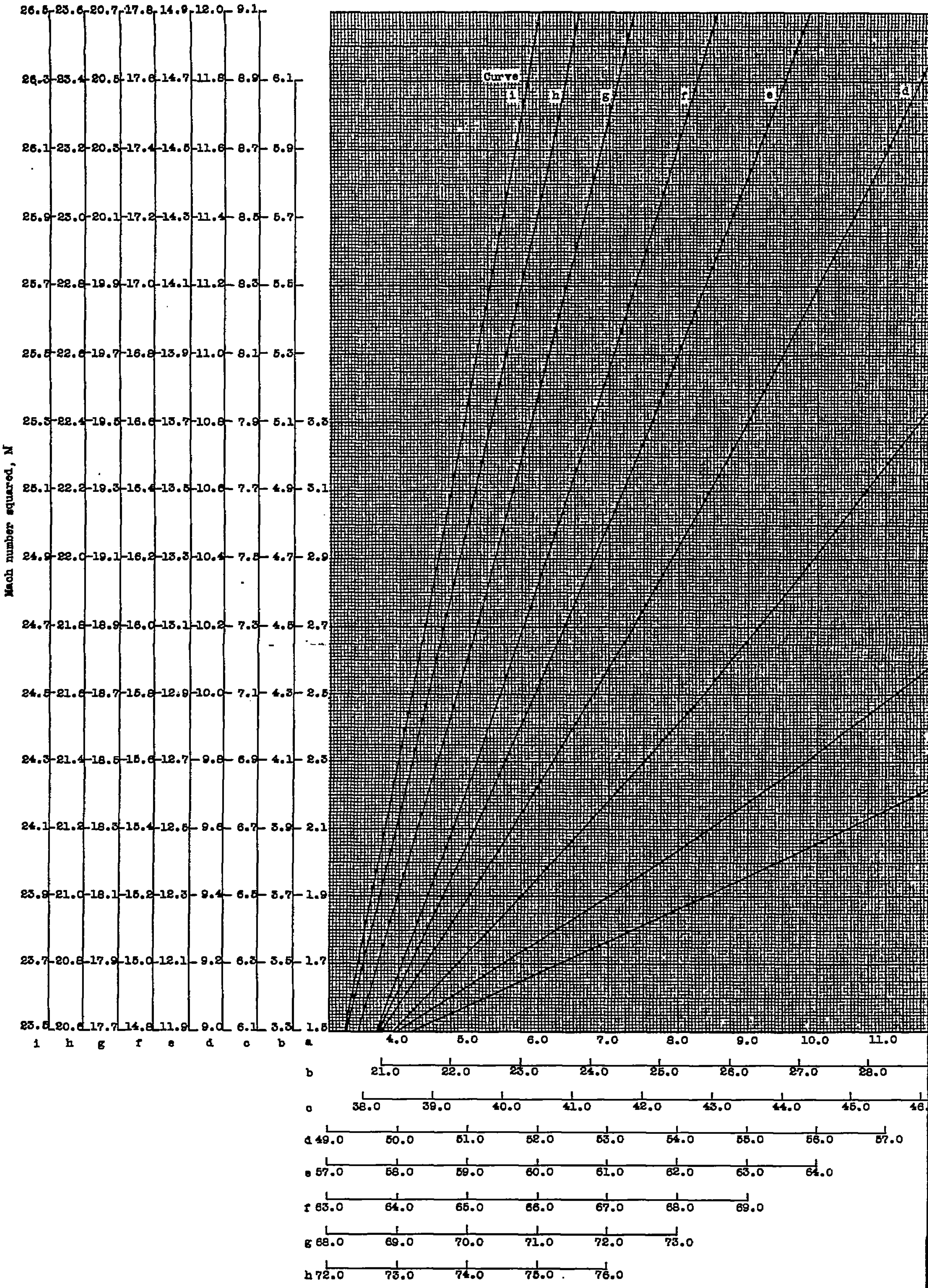
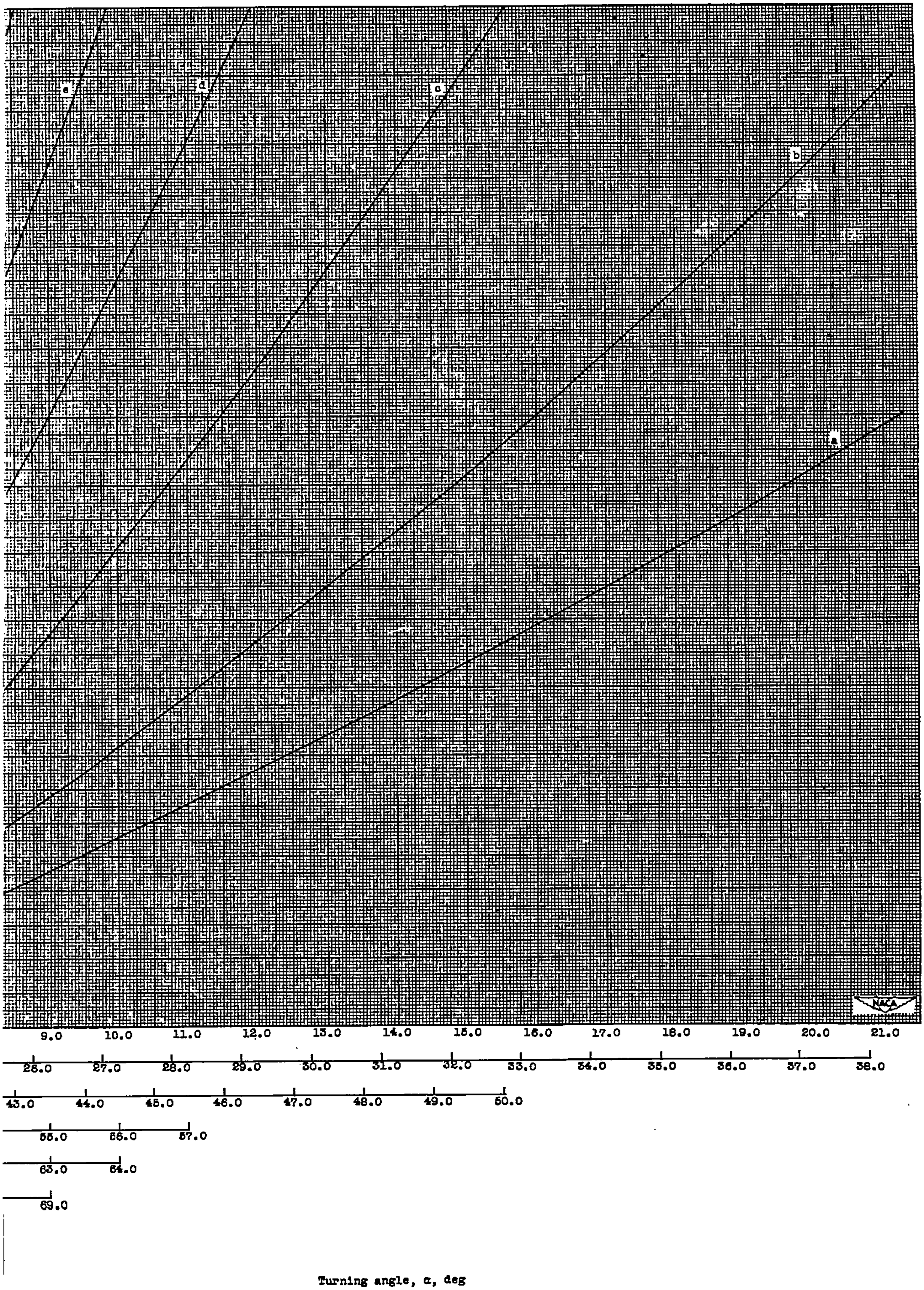


Figure 13. – Variation of Mach number s


e
d
c
b
a
NACA
9.0 10.0 11.0 12.0 13.0 14.0 15.0 16.0 17.0 18.0 19.0 20.0 21.0
26.0 27.0 28.0 29.0 30.0 31.0 32.0 33.0 34.0 35.0 36.0 37.0 38.0
43.0 44.0 45.0 46.0 47.0 48.0 49.0 50.0
55.0 56.0 57.0
63.0 64.0
69.0
Turning angle, α, deg